Putnam Σ.2

Po-Shen Loh

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1 Problems

Putnam 1985/B4. Let C be the unit circle $x^2+y^2=1$. A point P is chosen randomly on the circumference C and another point Q is chosen randomly from the interior of C (these points are chosen independently and uniformly over their domains). Let R be the rectangle with sides parallel to the x and y-axes with diagonal PQ. What is the probability that no point of R lies outside of C?

Putnam 1985/B5. Evaluate

$$\int_0^\infty t^{-1/2}e^{-1985(t+t^{-1})}dt.$$

You may assume that

$$\int_{-\infty}^{\infty} e^{-x^2}dx = \sqrt{\pi}.$$

Putnam 1985/B6. Let G be a finite set of real $n \times n$ matrices $\{M_i\}$, $1 \le i \le r$, which form a group under matrix multiplication. Suppose that $\sum_{i=1}^r \mathrm{tr}(M_i) = 0$, where $\mathrm{tr}(A)$ denotes the trace of the matrix A. Prove that $\sum_{i=1}^r M_i$ is the $n \times n$ zero matrix.