Putnam Σ.14

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1 Problems

Putnam 1991/B4. Suppose p is an odd prime. Prove that

$$\sum_{j=0}^{p} \binom{p}{j}\binom{p+j}{j} \equiv 2^p + 1 \pmod{p^2}.$$

Putnam 1991/B5. Let p be an odd prime and let $\mathbb{Z}_p$ denote (the field of) integers modulo p. How many elements are in the set

$$\{x^2 : x \in \mathbb{Z}_p\} \cap \{y^2 + 1 : y \in \mathbb{Z}_p\}?$$

Putnam 1991/B6. Let a and b be positive numbers. Find the largest number c, in terms of a and b, such that

$$a^x b^{1-x} \le a\frac{\sinh ux}{\sinh u} + b\frac{\sinh u(1-x)}{\sinh u}$$

for all u with $0 < |u| \le c$ and for all x, $0 < x < 1$. (Note: $\sinh u = (e^u - e^{-u})/2$.)