2. Number theory

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CMU Putnam Seminar, Fall 2012

1 Classical results

1. Show that $\sqrt{6}$ is irrational.

2 Problems

Putnam 1955/A1. Prove that if a, b, c are integers and $a\sqrt{2} + b\sqrt{3} + c = 0$, then $a = b = c = 0$.

Natural. Find all integral x and y satisfying the equation $2\sqrt{6} + 5\sqrt{10} = \sqrt{x} + \sqrt{y}$.

Putnam 1956/A2. Given any positive integer n, show that we can find a positive integer m such that mn uses all ten digits when written in the usual base 10.

Putnam 1954/B1. Show that for any positive integer r, we can find integers m, n such that $m^2 - n^2 = r^3$.

Putnam 1958/B2. Let n be a positive integer. Prove that $n(n+1)(n+2)(n+3)$ cannot be a square or a cube.

Putnam 1952/A6. Prove that there are only finitely many cuboidal blocks with integer sides $a \times b \times c$, such that if the block is painted on the outside and then cut into unit cubes, exactly half the cubes have no face painted.

Putnam 1959/B6. α and β are positive irrational numbers satisfying $1/\alpha + 1/\beta = 1$. Let $a_n = \lfloor n\alpha \rfloor$ and $b_n = \lfloor n\beta \rfloor$, for $n = 1, 2, 3, \ldots$. Show that the sequences a_n and b_n are disjoint and that every positive integer belongs to one or the other.

Putnam 1954/B6. If x is a positive rational, show that we can find distinct positive integers $a_1, a_2, \ldots, a_n$ such that $x = \sum 1/a_i$.

Putnam 1953/B7. Show that we can express any irrational number $0 < \alpha < 1$ uniquely in the form $\sum_1^\infty (-1)^{n+1} 1/(a_1 a_2 \cdots a_n)$, where a_i is a strictly monotonic increasing sequence of positive integers. Find a_1, a_2, a_3 for $\alpha = 1/\sqrt{2}$.

3 Homework

Please write up solutions to two of the problems, to turn in at next week's meeting. One of them may be a problem that we discussed in class.