Even more advanced Putnam training

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1 Problems

Putnam 2000/A1. Let A be a positive real number. What are the possible values of $\sum_{j=0}^{\infty} x_j^2$, given that $x_0, x_1, \ldots$ are positive numbers for which $\sum_{j=0}^{\infty} x_j = A$?

Putnam 2004/A2. Let T_1 be a triangle with side lengths a_1, b_1, c_1, and let T_2 be an **acute** triangle with side lengths a_2, b_2, c_2. Suppose that $a_1 \leq a_2$, $b_1 \leq b_2$, and $c_1 \leq c_2$. Does it follow that $\text{area}(T_1) \leq \text{area}(T_2)$?

Putnam 2004/A3. Define a sequence $u_0, u_1, u_2, \ldots$ by $u_0 = u_1 = u_2 = 1$, and thereafter by the condition

$$\det \begin{pmatrix} u_n & u_{n+1} \\ u_{n+2} & u_{n+3} \end{pmatrix} = n!$$

for all $n \geq 0$. Show that u_n is an integer for all n. (By convention, $0! = 1$.)