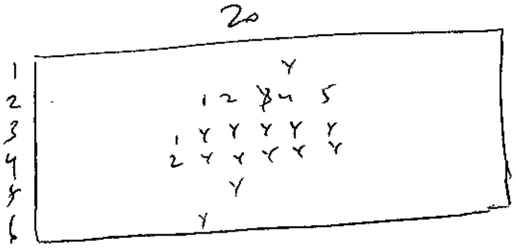
1996/A3

2010-11-08

(2)

2 same

20



$\binom{6}{3} = 20.$

$\frac{1}{2}$ each

$\binom{20}{5}\binom{6}{2}2\left(\frac{1}{2}\right)^{60}$ $\quad \frac{1}{2^9}$

N

bad.

N

N N N N N

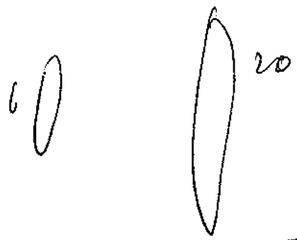
N N N N N

N

N

or

6 $\quad$ 20



1995/A2 $\quad \sqrt{\sqrt{x+a} - \sqrt{x}} \approx \sqrt{\frac{a}{2\sqrt{x}}} \simeq \frac{\sqrt{\frac{a}{2}}}{x^{\frac{1}{4}}} - \frac{\sqrt{\frac{b}{2}}}{x^{\frac{1}{4}}}$ $\quad$ diverge unless $a = b$.

$\sqrt{x+a} - \sqrt{x} =$

$f'' = -\frac{1}{4x^{3/2}}$

$\sqrt{x+a} = \sqrt{x} + \frac{a}{2\sqrt{x}} \pm \frac{4\text{ up to } a}{8x^{3/2}}$ $\quad$ Telescope if $a=b$.

$f(x) = \sqrt{\sqrt{x+a} - \sqrt{x}}$.

$\frac{a}{2\sqrt{x}} + O\left(\frac{1}{x^{3/2}}\right)$

$\int_a^L f(x) - f(x-a) = \int_a^L f(x) - \int_0^{L-a} f(x)$

$\int_a^\infty f(x) - f(x-a)$

$= \int_a^\infty f(x) - \int_0^\infty f(x)$.

$(1+o(1))\frac{a}{2\sqrt{x}}$ $\quad = \int_{L-a}^{L} f(x) - \int_0^a f(x)$ finite

(bad) $\times a$

$\approx \frac{a+b}{\sqrt{x}}$.

If $a \neq b$:

$$\frac{\left(\sqrt{x+a} - \sqrt{x}\right) - \left(\sqrt{x} - \sqrt{x-b}\right)}{\sqrt{\sqrt{x+a} - \sqrt{x}} + \sqrt{\sqrt{x} - \sqrt{x-b}}} = \frac{\sqrt{x+a} - 2\sqrt{x} + \sqrt{x-b}}{}$$

2010-11-08

(3)

$$\sqrt{x+a} - \sqrt{x} = \sqrt{x}\left[\sqrt{1+\frac{a}{x}} - 1\right],$$

$$= \sqrt{x}\left[\frac{a}{2x} + O\left(\frac{a}{x^2}\right)\right]$$

$$\sqrt{1+t} = 1 + \frac{t}{2} + O(t^2) \quad \text{for } t$$

$$(\quad)t^2$$

f''(somewhere $1 \dots t$)

$f = \sqrt{x}$.

$$\sqrt{\sqrt{x+a} - \sqrt{x}} = \quad \sqrt{x}\,(1-o(1))\frac{a}{2x} = \underbrace{(1-o(1))}\frac{a}{2\sqrt{x}}.$$

approaches 1 eventually

$$\sqrt{\frac{a}{2}}\;\frac{1}{x^{\frac{1}{4}}},$$

Subtract ... Eventually factors are s.t. $(1-o(1))\sqrt{\frac{a}{2}}$ and $(1-o(1))\sqrt{\frac{b}{2}}$ have gap, sustained. $\int \frac{1}{x^{\frac{1}{4}}} \nearrow \infty$.

1988/B1

$$\frac{\left(x+\frac{1}{x}\right)^6 - \left(x^6 + \frac{1}{x^6}\right) - 2}{\left(x+\frac{1}{x}\right)^3 + \left(x^3 + \frac{1}{x^3}\right)}.$$

$$x=1: \quad \frac{2^6 - 2 - 2}{2^3 + 2} = \frac{60}{10} = 6$$

$$\left(x^3 + \frac{1}{x^3}\right)^2.$$

$$\frac{\left[\left(x+\frac{1}{x}\right)^3\right]^2 - \left(x^3 + \frac{1}{x^3}\right)^2}{\text{sum}} = \left(x+\frac{1}{x}\right)^3 - \left[x^3 + \frac{1}{x^3}\right]$$

$$= 3x + \frac{3}{x}. \quad \text{min @ } 6.$$

$$= 6\left(\frac{x+\frac{1}{x}}{2}\right) \underset{\text{AM GM}}{\geq} 6.$$