

# Even more advanced Putnam training

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## 1 Problems

**Putnam 2006/B1.** Show that the curve  $x^3 + 3xy + y^3 = 1$  contains only one set of three distinct points,  $A$ ,  $B$ , and  $C$ , which are vertices of an equilateral triangle, and find its area.

**Putnam 2007/A2.** Find the least possible area of a convex set in the plane which touches both branches of the hyperbola  $xy = 1$  and both branches of  $xy = -1$ .

**Putnam 2005/A3.** Let  $p(z)$  be a nonconstant polynomial of degree  $n$ , all of whose zeros have absolute value 1 in the complex plane. Put  $g(z) = p(z)/z^{n/2}$ , where we have chosen the positive branch of the square root. Show that all zeros of  $g'(z) = 0$  have absolute value 1.