

Even more advanced Putnam training

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1 Problems

Putnam 1996/B1. Define a **selfish** set to be a set which has its own cardinality (number of elements) as an element. Find, with proof, the number of subsets of $\{1, 2, \dots, n\}$ which are *minimal* selfish sets, that is, selfish sets none of whose proper subsets is selfish.

Putnam 1998/A2. Let s be any arc of the unit circle lying entirely in the first quadrant. Let A be the area of the region lying below s and above the x -axis, and let B be the area of the region lying to the right of the y -axis and to the left of s . Prove that $A + B$ depends only on the arc length, and not on the position, of s .

Putnam 2002/B3. Show that for all integers $n > 1$,

$$\frac{1}{2ne} < \frac{1}{e} - \left(1 - \frac{1}{n}\right)^n < \frac{1}{ne}.$$