Even more advanced Putnam training

Po-Shen Loh

6 October 2010

1 Problems

Putnam 2005/A1. Show that every positive integer is a sum of one or more numbers of the form $2^r 3^s$, where r and s are nonnegative integers and no summand divides another. (For example, $23 = 9 + 8 + 6$.)

Putnam 2001/A2. You have coins $C_1, C_2, \ldots, C_n$. For each k, C_k is biased so that, when tossed, it has probability $\frac{1}{2k+1}$ of falling heads. If the n coins are tossed, what is the probability that the number of heads is odd? Express your answer as a rational function of n.

Putnam 2003/B3. Show that for each positive integer n,

$$n! = \prod_{i=1}^{n} \text{lcm}\{1, 2, \ldots, \lfloor n/i \rfloor\}.$$

(Here, lcm denotes the least common multiple, and $\lfloor x \rfloor$ denotes the greatest integer $\leq x$.