..., 1, 1, 1, 2, 3, ..., 2005, 2006 | 2007, 2009, 2013, 2016, ...

identity | $x_{k+1} = x_k + x_{k-2005}$

Need 2005 canceled terms $\downarrow\downarrow$ by 2006.

Residues for 2005 cancel if same?

#=2005   # = 2005.

$\leftarrow$ —————— doubly ∞ —————————— $\rightarrow$
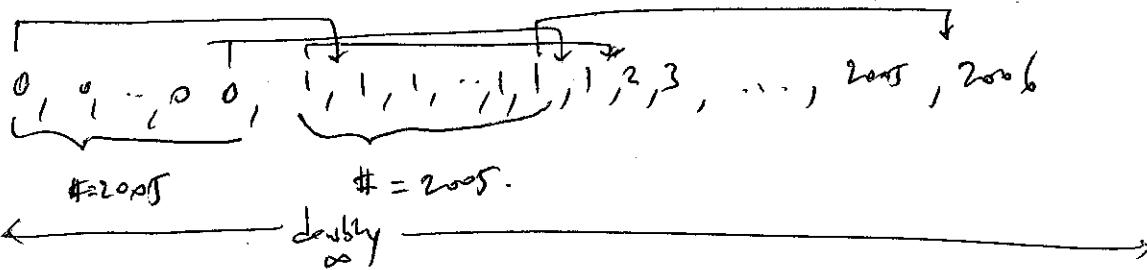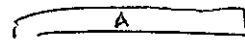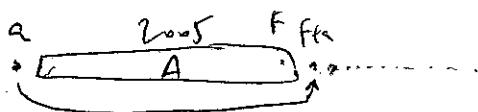
~~So segnets of 2005~~

Consider segnets of 3000. There must be repeat since ∞ long residues.
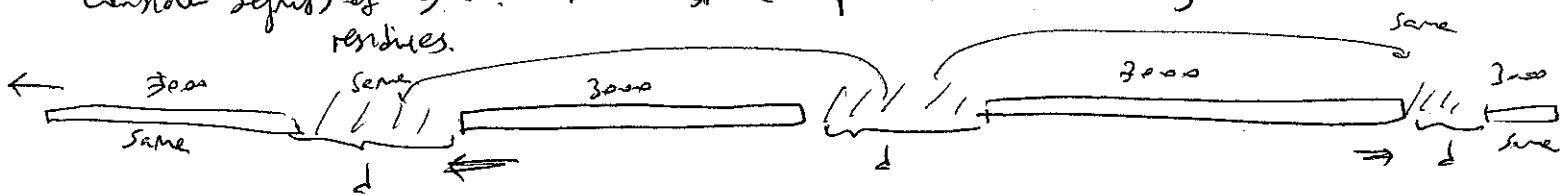
So the 0-pattern must recur.

2006/B1. $x^3 + 3xy + y^3 = 1$,

$x^3 + 3xy + y^3 - 1 = 0.$

$(x+y)(x^2 - xy + y^2) + 3xy - 1 = 0.$

$(x+y)(x+y)$

$x = 0: y = 1.$

$x = 1: \cancel{y} 3y + y^3 - 1 = 0,$

$(a_2 x^2 + a_1 x + a_0)(c_2 y^2 + c_1 y + c_0) + (b_2 x^2 + b_1 x + b_0)(d_2 x^2 + d_1 x + d_0)$

$a_2 c_2 + b_2 d_2 = 1,$
$a_2 c_0 + b_2 d_0 = 0,$ $\rightarrow$ $-\frac{a_2}{b_2} = \frac{d_0}{c_0} = \frac{d_1}{c_1}$ $a_1 c_0 + b_1 d_0 = 0$
$a_2 c_1 + b_2 d_1 = 0$ $x = -\frac{a_1}{b_1} = \frac{d_2}{c_2}$ ✳ $a_1 c_2 + b_1 d_2 = 0.$

2005/B1

$1 + xy + x^2 y^2 = a(x) c(y) + b(x) d(y).$

$0 = a'''(x) c(y) + b'''(x) d(y).$

Sin c, d deg ≤ 2

So

If $a'''(x) \ne 0$, $b'''(x) \ne 0$,
then c, d are scalar $\ne$ just ac.
If one 0, other not, th $d \equiv 0$, just ac.
If both 0, ok.