Even more advanced Putnam training

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Solution:

1 Announcements

- Wed next week
- CMU has only ever had max 2 in the HM.
- Scores required for HM, around the top 75 students.
- In order to make rank 70:

year	score
2009	53
2008	60
2007	48
2006	56
2005	49

- Suggested score breakdown

Q	pts
1	10
2	10
3	10
4	1
5	1
6	1
tot	33

So if only solve 5 questions, get 56 points

2 Problems

Putnam 2001/A1. Consider a set S and a binary operation $*$, i.e., for each $a, b \in S$, $a * b \in S$. Assume $(a * b) * a = b$ for all $a, b \in S$. Prove that $a * (b * a) = b$ for all $a, b \in S$.

Solution: FIX

Putnam 2001/B2. Find all pairs of real numbers (x, y) satisfying the system of equations

$$\frac{1}{x} + \frac{1}{2y} = (x^2 + 3y^2)(3x^2 + y^2)$$
$$\frac{1}{x} - \frac{1}{2y} = 2(y^4 - x^4)$$

Solution: FIX

Putnam 2001/A3. For each integer m, consider the polynomial

$$P_m(x) = x^4 - (2m+4)x^2 + (m-2)^2.$$

For what values of m is $P_m(x)$ the product of two nonconstant polynomials with integer coefficients?

Solution: FIX