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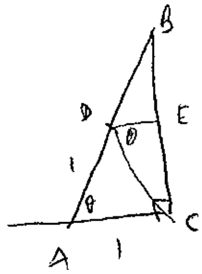
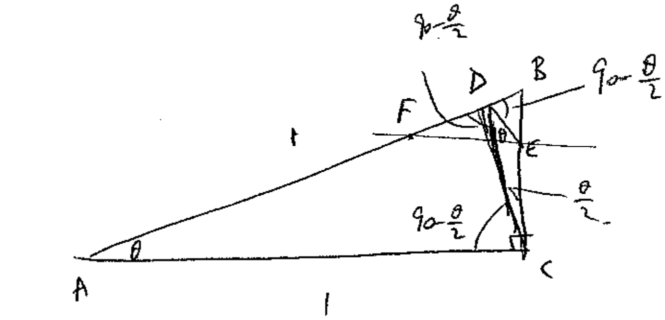
1

1999/B1





$$\frac{BE}{\cos\frac{\theta}{2}} = \frac{DE}{\cos\theta} = \frac{EC}{\sin\theta}\cdot\frac{\sin\frac{\theta}{2}}{\cos\theta}$$

$$\frac{BE}{BE+EC}\times 1.$$

recip: $1+\frac{EC}{BE}.$

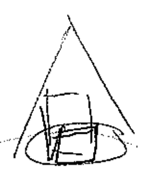
$$\frac{DE}{\sin\frac{\theta}{2}} = \frac{EC}{\sin\theta}.$$

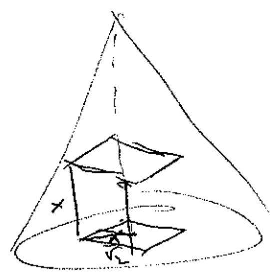
$$\frac{EC}{BE} = \frac{\sin\theta\cos\theta \to 1}{\sin\frac{\theta}{2}\cos\frac{\theta}{2} \to 1}.$$

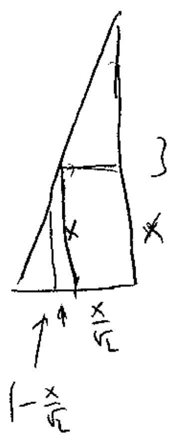
$\to 2.$

$\to \frac{1}{3}.$

1998/A1







$$\frac{1-\frac{x}{\sqrt{2}}}{x} = \frac{1}{3}$$

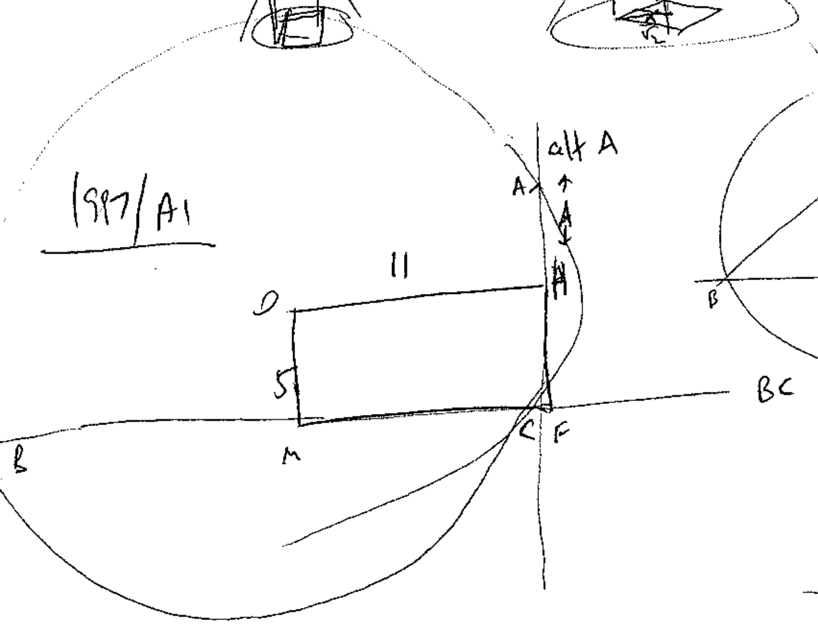
$$3-\frac{3}{\sqrt{2}}x = x.$$

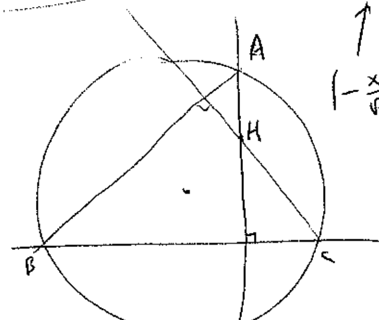
$$3 = \left(\frac{3}{\sqrt{2}}+1\right)x$$

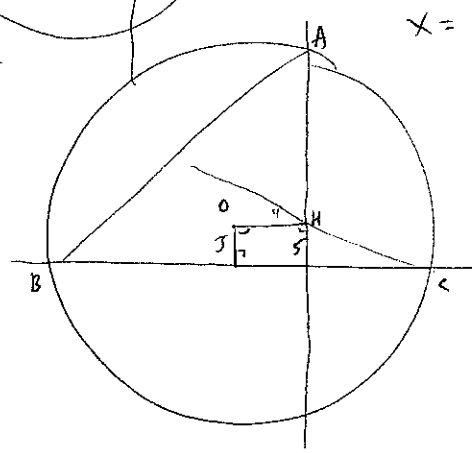
$$= \left(\frac{3+\sqrt{2}}{\sqrt{2}}\right)x.$$

$$x = \frac{3\sqrt{2}}{3+\sqrt{2}} = \frac{(3\sqrt{2})(3-\sqrt{2})}{9-2} = \frac{9\sqrt{2}-6}{7}.$$

1997/A1







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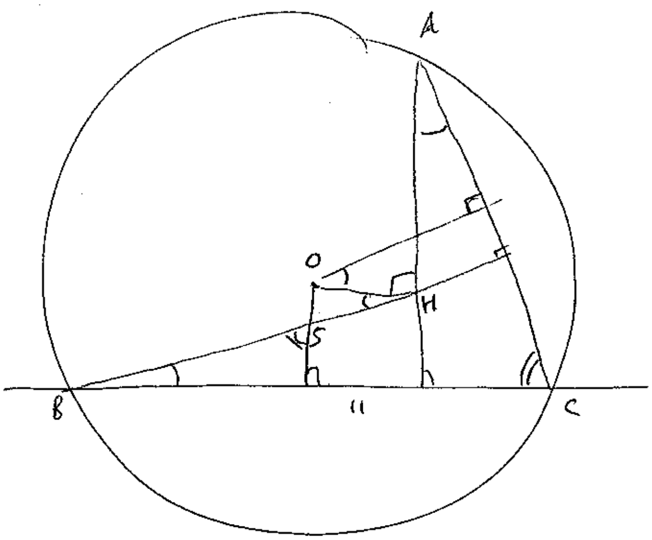
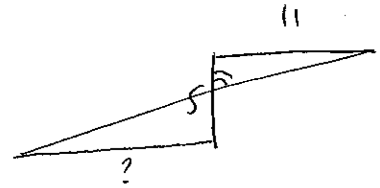
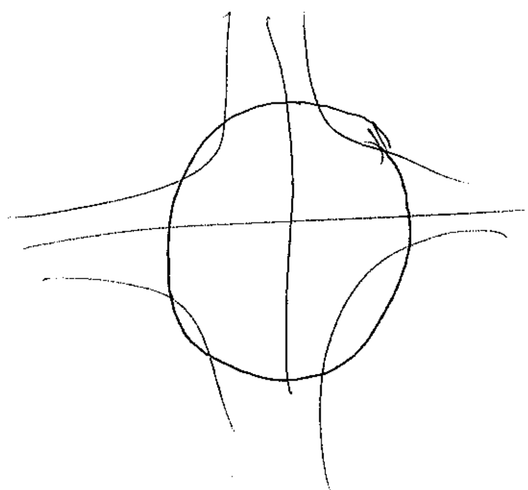
(2)

1997/A1

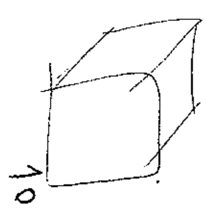




GA 597



GA 632



ortho~~normal~~gonal same norm

(Q)

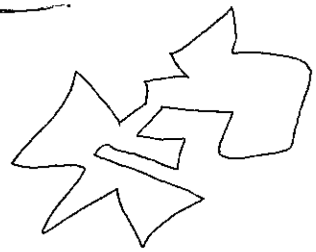
$$A = \begin{array}{c} \begin{bmatrix} \text{---} \\ \text{---} \\ \text{---} \end{bmatrix} \end{array} \begin{array}{l} \leftarrow \text{cards 1} \\ \leftarrow \text{cards 2} \\ \leftarrow \text{cards 3} \end{array}$$

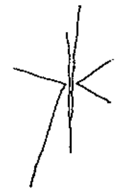
$$AA^T = \begin{bmatrix} k & & & 0 \\ & k & & \\ & & k & \\ 0 & & & k \end{bmatrix} = kI.$$

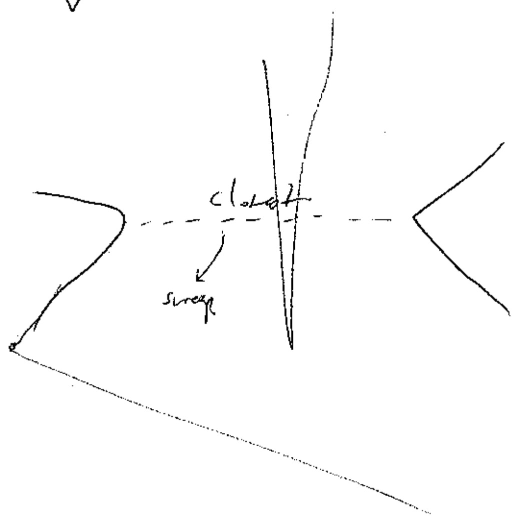
$$(\det A)^2 = k^{\text{odd}} \Rightarrow \underline{k}^{\text{odd}} \text{ is a square} \Rightarrow k \text{ is square}$$

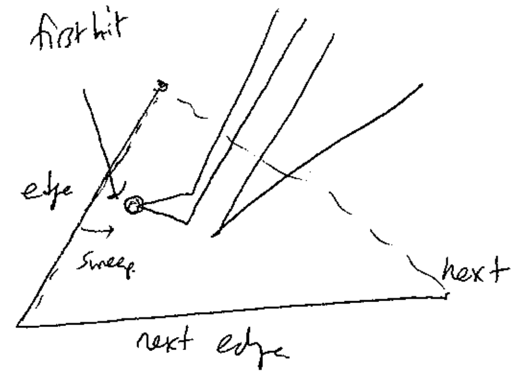
$\Rightarrow$ integer edge length!

Triangulation

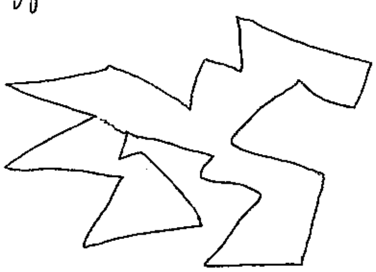








(L) Every polygon has an interior chord.



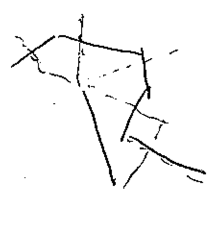
do each separately

n=3 is trivial

(L) Can 3-color vtxs of triangulation: segments connecting vtxs s.t. all ~~pieces~~ faces are triangles

Induction.

Dual is tree



(L) Every polygon has "ear".

∃ has triangulation. Clip ears ~~to~~.

So ~~Any~~ triangulation made up of adding ears back on, and can 3-color vtxs

Choose minimal color class $\left\lfloor \frac{n}{3} \right\rfloor$ Is each △ covered? Yes, has one of each color.

Tight:

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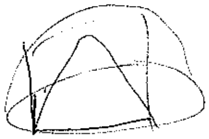
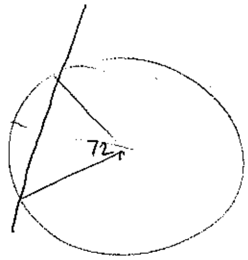
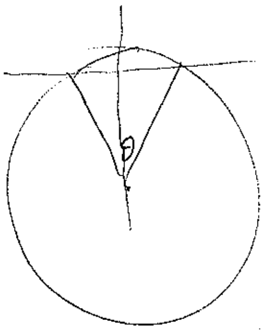
④

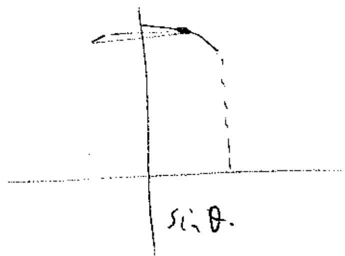
1998/B3

cut off spherical cap.

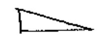


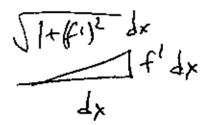






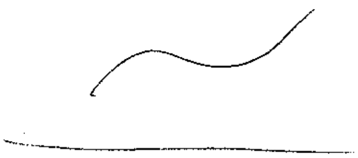
f'



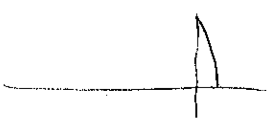


$\int 2\pi x$

$f(x) = \sqrt{1-x^2}$.



$\int 2\pi y \sqrt{1+(y')^2}\,dx$.



$y = \sqrt{1-x^2}$.

$y' = \frac{-2x}{2\sqrt{1-x^2}} = -\frac{x}{\sqrt{1-x^2}}$.

$\cos\frac{\theta}{2} \cdots 1$.

$\int_{\cos\frac{\theta}{2}}^{1} 2\pi \sqrt{1-x^2}\sqrt{1+\frac{x^2}{1-x^2}}$

cut off:

$= \int_{\cos\frac{\theta}{2}}^{1} 2\pi \sqrt{1-x^2}\sqrt{\frac{1}{1-x^2}} = 2\pi\left(1-\cos\frac{\theta}{2}\right)$

$\frac{1}{2}\left(5 \times 2\pi\left(1-\cos\frac{72}{2}\right)\right)$.

$2\pi - 5\pi\left(1-\cos\frac{72^\circ}{2}\right) = 5\pi\cos 36^\circ - 3\pi$

2004/B4

$f(z) = z_0 + (z-z_0)e^{i\theta}$

$z_1 + \left(z_0 + (z-z_0)e^{i\theta} - z_1\right)e^{i\phi}$

$= \left[z_1 + (z_0-z_1)e^{i\phi}\right] + (z-z_0)e^{i(\theta+\phi)}$

$\alpha + ze^{i\theta}$.

is $z_0 + (z-z_0)e^{i\theta}$.

$\underbrace{z_0 - z_0(e^{i\theta}) = \alpha}_{\text{given}}$, solve for z_0.

2004/B4

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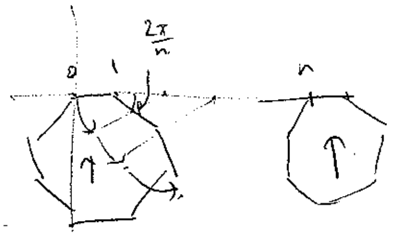
①



$z \mapsto z_0 + (z - z_0)e^{i\theta}$

$z \mapsto 1 + (z-1)e^{i\theta}$ 1st

$\mapsto 2 + \left[1 + (z-1)e^{i\theta} - 2\right]e^{i\theta}$

$= 2 + \left[(z-1)e^{i\theta} - 1\right]e^{i\theta}$

$= 2 + (z-1)e^{2i\theta} - e^{i\theta}$ 2nd

$\mapsto 3 + \left[(z-1)e^{2i\theta} - e^{i\theta} - 1\right]e^{i\theta}$

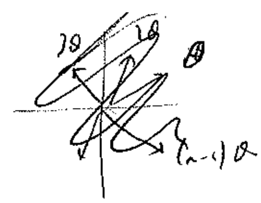
$= 3 + (z-1)e^{3i\theta} - e^{2i\theta} - e^{i\theta}$

$\mapsto 4 + \left[z e^{3i\theta} - e^{3i\theta} - e^{2i\theta} - e^{i\theta} - 1\right]e^{i\theta}$

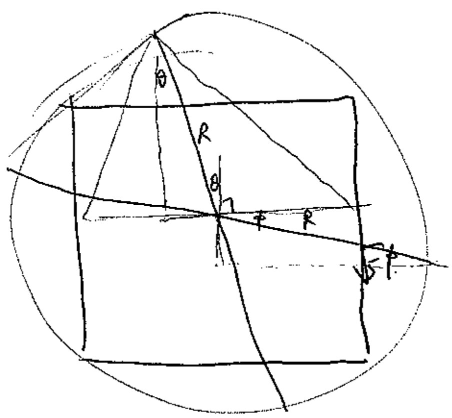
$\mapsto$
$\vdots$

$\mapsto n + z\underbrace{e^{ni\theta}}_{1} \underbrace{- e^{ni\theta} - e^{(n-1)i\theta} - e^{(n-2)i\theta} - \cdots - e^{i\theta}}_{0}$

$\left(e^{i\theta} + \cdots + e^{ni\theta}\right)\left(e^{i\theta} - 1\right) = 0$



2000/A3



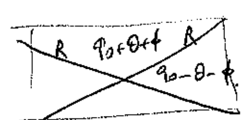
diam.

$R = \frac{\sqrt{5}}{\sqrt{2}}$

(max sum of 4 altitudes.

$\left[R(\cos\theta + \cos\phi) - \frac{\sqrt{5}}{2}\times 2 \right] 2) \times \frac{1}{2}\text{base} + 5$ $\sqrt{5}$ sqr.

s.t.



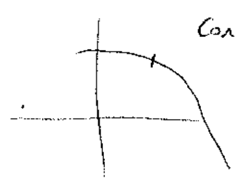
$R^2\left(\sin(90+\theta+\phi) + \sin(90-\theta-\phi)\right)$

$= R^2\left(2\cos(\theta+\phi)\right)$

$\frac{25}{4}\cos(\theta+\phi) = 4.$

Maximize $\sqrt{\frac{5}{2}}\left(\cos\theta + \cos\phi\right)$

max at equality



Concave.

$\cos(\theta+\phi) = \frac{4}{5}.$

$\cos 2\theta = \frac{4}{5}.$

$2\cos^2\theta - 1 = \frac{4}{5}.$

$\cos^2\theta = \frac{9}{10}.$

$\sqrt{\frac{5}{2}}\, 2\cos\theta \longrightarrow \sqrt{\frac{5}{2}}\, 2\sqrt{\frac{9}{10}}$

$= \frac{6}{2} = 3$

$(3-\sqrt{5})\, 2 \times \frac{\sqrt{5}}{2} + 5.$

$= 3\sqrt{5} - 5 + 5 \longrightarrow 3\sqrt{5}$