Discrete Mathematics 21-228: Summer II 2000
Final Exam
Due Thursday, 8/10 by **1:00pm**

August 9, 2000

You must complete this exam on your own. Absolutely no collaboration is permitted. You may not use any sources other than the notes and texts listed on the course page. You may use a calculator or a computer algebra system to help simplify answers.

Attempt any **3 of the following 4** problems. If you attempt all 4, **your best score will be ignored**.

A recurrence of a different kind

Your goal is to show

$$1+\frac{1}{1+\frac{1}{1+\frac{1}{1+\ldots}}}=\sqrt{1+\sqrt{1+\sqrt{1+\ldots}}}=\frac{1+\sqrt{5}}{2}.$$

You may assume each expression above converges. Here's a suggested strategy.

- Show $\frac{1+\sqrt{5}}{2}=\phi=\frac{1}{1+\frac{1}{1+\ldots}}$ by finding an equation ϕ must satisfy.
- Show $\frac{1+\sqrt{5}}{2}=\phi=\sqrt{1+\sqrt{1+\sqrt{1+\ldots}}}$ in a similar manner as above.

A farewell to D(iscrete)M(ath)

Let a_n be the number of words of length n over the alphabet $\{A,B,C,\ldots,Z\}$ which do not contain the subword DM. Show that

$$a_n=26a_{n-1}-a_{n-2};$$

determine suitable initial conditions and solve the recurrence.

Take this permutation, turn it sideways, and...

Let p_n be the number of permutations of the multiset $\{1,1,2,2,\ldots,n,n\}$ which do not contain consecutive $i's$ for $1\le i\le n$. We have $p_2=2$ (1212 and 2121 are the only valid permutations) for instance. Show that

$$p_n = \sum_{i=0}^{n} (-1)^i \binom{n}{i} \frac{(2n-i)!}{2^{n-i}}.$$

YAPP

This is Yet Another Party Problem. A couple invites $n-1$ other couples for a little shindig. As this is no usual party, instead of shaking hands people arm-wrestle to introduce themselves. Each person arm-wrestles any other person at most once, and a person cannot arm-wrestle himself (take my word on this one). In addition, no one arm-wrestles her spouse (presumably, they've already done it). After every one has passed out from sheer exhaustion, the hostess asks each person, including her spouse, how many times he's arm-wrestled that night. Surprisingly enough, all $2n-1$ responses are distinct. The question is, how many times did the hostess and her spouse each arm-wrestle during the party? A viable strategy is to guess an answer by considering small values of n and prove your conjecture by induction.