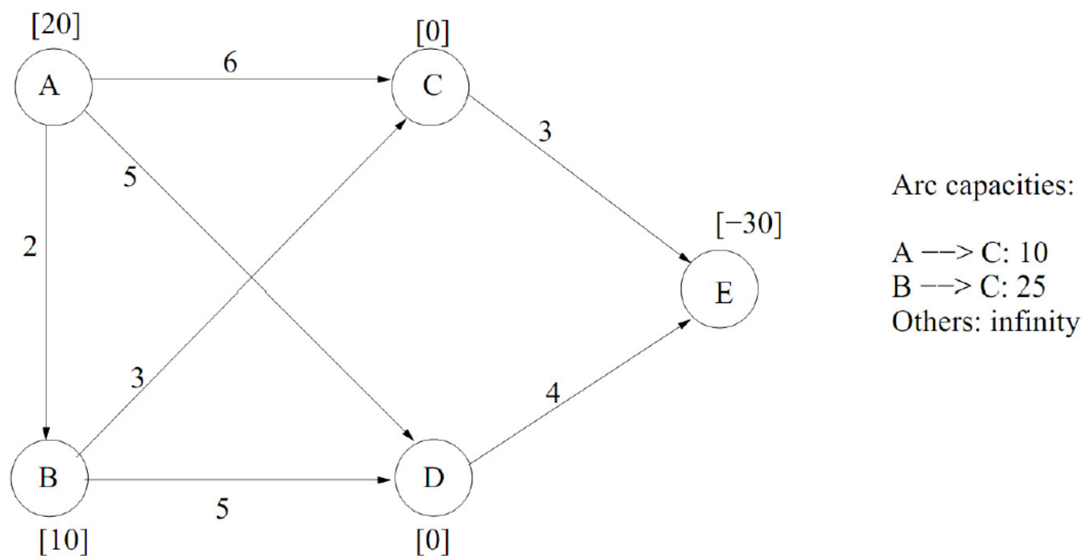
21-292 Midterm II
Professor Shlomo Ta'asan

Problem 1. (34 points) Consider the following network problem. Start with the initial solution in which X_{BC} reaches its upper bound and other basic variables are the arcs A→ D, D→ E, A → B, C→ E.
(a) Update the network to reflect the upper bound for X_{BC} and perform the network simplex method to get an optimal solution.
(b) Is the optimal solution unique? If not find another solution.



Problem 2. (33 points) Consider the following transportation problem

Min 8 X11 + 6 X12 + 8 X13 + 7 X21 + 4 X22 + 5 X23 + 10 X31 + 9 X32 + 9 X33
subject to

X11 + X12 + X13 = 20
X21 + X22 + X23 = 20
X31 + X32 + X33 = 15
X11 + X21 + X31 = 15
X12 + X22 + X32 = 15
X13 + X23 + X33 = 25

(a) Construct a table for of this problem and solve it using the transportation simplex method starting with the North-West initial solution.
(b) Is the solution unique? If not find another solution.

Problem 3 (33 pts) Consider the following problem.

Maximize $Z = 3x_1 + 4x_2 + 8x_3$,

subject to

$2x_1 + 3x_2 + 5x_3 \leq 9$

$x_1 + 2x_2 + 3x_3 \leq 5$

and

$x_1 \geq 0, \; x_2 \geq 0, \; x_3 \geq 0.$

Let x_4 and x_5 denote the slack variables for the respective functional constraints. After we apply the simplex method, the final simplex tableau is.

Basic Variable	Eq.	Coefficient of: Z	x_1	x_2	x_3	x_4	x_5	Right Side
Z	(0)	1	0	1	0	1	1	14
x_1	(1)	0	1	-1	0	3	-5	2
x_3	(2)	0	0	1	1	-1	2	1

Let the right hand size of the first constraint change from $b_1 = 9$ to $9 + \theta$. (i) Find the range of θ for which the set of basic variables are unchanged. (ii) Find the solution for $b_1 = 12$. (iii) The value of $c_2 = 4$ is changed to $4 + x$. for what range of x the solution is still optimal? (iv) The value of $c_3 = 8$ is changed to $8 + y$. For what values of y, the current solution is still optimal?

Problem 4 (5 points) Consider the following problem

Min $c_1 x_1 + c_2 x_2 + \ldots + c_n x_n$

subject to

$x_1 + x_2 + \ldots + x_n = 1 \quad x_j >= 0, \quad j=1,\ldots,n$

(a) find an optimal solution and prove its optimality.

(b) Is the solution unique? Explain your answer.