## Voting Methods

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## **1** Basics of Preferential Voting

In a preferential voting system, we work within the following framework:

- $\bullet\,$  We have a finite set of candidates  ${\cal C}$
- We have a finite set of voters X
- Each voter  $x \in X$  indicates an ordinal preference on  $\mathcal{C}$ , represented as a total linear order  $\leq_x$

Our goal is to use the collection  $\{\leq_x | x \in X\}$  to construct a total order  $\leq$  on C. Below are some common methods for so doing. As it will frequently be relevant, we write  $\max_x$  and  $\min_x$  to denote the maximum and minimum for an individual voter x, and  $\max$  and  $\min$  to denote the maximum and minimum for the total order  $\leq$ .

- 1. Plurality Voting. Candidates are ranked by the number of first place votes they receive. That is, we set  $A \leq B$  if  $|\{x \in X | A = \max_x\}| \leq |\{x \in X | B = \max_x\}|$ .
- 2. Runoff Voting (for just 1 winner). This method occurs in two phases. In the first phase, we select two candidates A and B by taking the top two ranked candidates in the Plurality method (i.e., those with the most number 1 votes). Then we select the winner of the election by considering the relative ranking between those two: if more voters place A above B, then A wins. Examples include: any US state or district with "jungle primaries" (Louisiana, Georgia, California, Alabama, etc.), French national elections.
- 3. Instant Runoff (Hare method). There are many variations on the instant runoff system. The most popular version is called the Hare voting method. The Hare method is used for voting in Australia and Ireland. In the first round, the number of first place votes for each candidate is tabulated. Then, the candidate having the least first place votes is put in last place, and removed from the voting matrix. This process is repeated until only one candidate remains. Here's an example.

We end with the ranking  $D \leq C \leq B \leq A$ .

4. Instant Runoff (Coomb's method, aka Reverse Hare). In a variant on the Hare method, called the Coomb's method, at each stage we eliminate the candidate with the most last place votes, rather than the candidate with the fewest first place votes. In that case, this same vote would proceed as follows.

We end with the ranking  $B \leq C \leq D \leq A$ . Notice that although our winner here did not change, we see a big difference in the overall ranking structure. Where B used to be in second place, by using the Coomb's method B was pushed down to last place.

5. Borda Count. In a Borda Count, the ranking of candidates by the voters is a way of assigning each candidate points. The candidate ultimately with the most points will be the one who wins the election. Generally, if there are n candidates, then each voter assigns points as follows: n points for first place, n-1 points for second place, etc. This kind of ranking system is used by professional golf and tennis associations to rank players.

Here's an example.

In this voting matrix, we calculate the score for each candidate.

score(A) = 10 \* 4 + 6 \* 3 + 5 \* 2 + 4 \* 3 + 2 \* 4 = 88 score(B) = 10 \* 3 + 6 \* 4 + 5 \* 1 + 4 \* 1 + 2 \* 1 = 65 score(C) = 10 \* 1 + 6 \* 2 + 5 \* 4 + 4 \* 2 + 2 \* 2 = 54score(D) = 10 \* 2 + 6 \* 1 + 5 \* 3 + 4 \* 4 + 2 \* 3 = 63

Hence, we rank candidates as  $C \leq D \leq B \leq A$ .

6. **Condorcet Method.** The Condorcet Method is a way of determining a winner by running something like a "round robin" contest. Each candidate is pitted against each other candidate individually. If there is a candidate who always beats every other, that candidate is declared the winner. If not, then points are assigned as follows: each win is worth 1 point, and each tie is worth 0.5 points. The candidate with the most points then wins. Here's an example.

If we take the head to head races we have:

$$A > B, A > C, A > D, B > C, B > D, D > C$$

Thus, we have that A gets 3 points, B gets 2 points, C gets 0 points, and D gets 1 point. We thus rank the candidates as  $C \le D \le B \le A$ .