Math 301 Homework

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Due 10 November 2017, 2:00 pm

Complete the following problems. Fully justify each response.

1. Let $G = G(A, B)$, with $|A| \leq |B|$. Define

$$k = \max\{|S| - |N(S)| \mid S \subset A\}.$$

   Show that a maximum matching in G is of size $|A| - k$.

2. Show that every tree has at most one perfect matching.

3. Use Tutte's Theorem to show that every bridgeless cubic graph has a perfect matching.

4. Suppose you are running the Gale-Shapley algorithm on the university admittance problem, in which the students ask universities for entry, and the universities then select the students they most desire. In this case, however, each university u_i has a capacity c_i of students they can admit. Modify the Gale-Shapley algorithm for this situation, and show that your modification will produce a stable matching.