Math 301 Homework

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Complete the following problems. Fully justify each response.

1. Determine which of the 5 voting methods discussed in class (Plurality, Hare, Coomb's, Condorcet, Borda Count) satisfy monotonicity. Prove that your answers are correct.

2. The Minimax Condorcet method is defined as follows:

   For any two candidates A and B, define $score(A, B) = d(A, B) - d(B, A)$, where $d(A, B)$ represents the number of voters ranking A above B. That is to say, $score(A, B)$ represents the margin between A and B, and is positive if and only if A beats B.

   We define the winner of the vote to be $\arg\min_X(\max_Y score(Y, X))$.

   (a) What is this method doing? Explain it.

   (b) Show that this method satisfies the Condorcet Criterion.

   (c) Suppose we rephrased the Condorcet Criterion as follows: If A loses every head-to-head competition, then A should lose. (This is called the Condorcet Loser Criterion). Show that the Minimax Condorcet Method fails the Condorcet Loser Criterion.

3. Prove that a Single Transferable Vote system, when used in an election with only one winner, is the same as an instant runoff using Hare Method.