## Math 301: Homework 7

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Complete the following problems. Fully justify each response.

- 1. Prove that if G is a k-regular bipartite graph, then the edges of G can be partitioned into disjoint sets  $M_1, M_2, \ldots, M_k$ , where each  $M_k$  is a perfect matching in G.
- 2. Let T be a tree.
  - (a) Prove that T has at most one perfect matching.
  - (b) Prove that T has exactly one perfect matching if and only if for every vertex  $v \in V(T)$ , the number of odd-sized components of  $T \setminus \{v\}$  is 1.
- 3. A bridge in a graph G is an edge e such that  $G \setminus \{e\}$  has more components than G. A graph is bridgeless if it has no bridges.

Let G be a bridgeless, 3-regular graph. Prove that G contains a perfect matching.