Math 301: Exam 1 25 September 2015

Turn off and put away your cell phone. No notes or books are permitted during this exam. No calculators or any other devices are permitted during this exam. Read each question carefully, answer each question completely, and show all of your work. Write solutions clearly and legibly; credit will not be given for illegible solutions. If any question is not clear, ask for clarification.

#	Points	Score
1	20	
2	20	
3	20	
Σ	60	

1. You have encountered a race of aliens! Each alien has three arms, which are distinguished as left, right and middle.

You have a party for n aliens, and supply them all with gloves. Supposing you have n triples of gloves, each of a different color, how many ways can you distribute these to the aliens so that no alien has a matched set of gloves?

- 2. Let  $k \ge 0$ . Define  $a_n$  to be the number of subsets S of [n] such that the distance between any two elements in S is at least k.
  - (a) Determine a recurrence relation for  $a_n$ . Include the initialization.
  - (b) Find the ordinary generating function for  $(a_n)$ , in closed form (i.e., without a summation).

- 3. Let n, s be positive integers.
  - (a) Prove the following binomial identity:

$$\sum_{k=1}^{n} \binom{n}{n-k} \binom{s-1}{k-1} = \binom{n+s-1}{n-1}.$$

(b) Find a *closed* expression (i.e., having no summation) for

$$\sum_{k=1}^{n} \binom{n}{k} \binom{s}{k} k.$$