

Math 301: Homework 8

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due 9 Nov 2015

Complete the following problems. Fully justify each response.

1. Prove that if G is a k -regular bipartite graph, then the edges of G can be partitioned into disjoint sets M_1, M_2, \dots, M_k , where each M_k is a perfect matching in G .
2. Let T be a tree.
 - (a) Prove that T has at most one perfect matching.
 - (b) Prove that T has exactly one perfect matching if and only if for every vertex $v \in V(T)$, the number of odd-sized components of $T \setminus \{v\}$ is 1.
3. A *bridge* in a graph G is an edge e such that $G \setminus \{e\}$ has more components than G . A graph is *bridgeless* if it has no bridges.

Let G be a bridgeless, 3-regular graph. Prove that G contains a perfect matching.