## Math 301: Homework 3

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Complete the following problems. Fully justify each response.

- 1. Let  $a_n$  be the number of paths of length n from (0,0) using steps of the form (1,0), (0,1), or (-1,0) and never intersecting themselves. (Note: these are different from our previous lattice paths in that we can take horizontal steps either left or right). Use a linear recurrence relation to find a closed form for  $a_n$ .
- 2. Find an ordinary generating function for  $(a_n)$  for each of the following recurrence relations:
  - $a_{n+1} = \alpha a_n + \beta$  for  $n \ge 0, a_0 = 0$ .
  - $a_{n+2} = 2a_{n+1} a_n$  for  $n \ge 0$ ,  $a_0 = 0$ ,  $a_1 = 1$
- 3. For even n, let  $e_n$  be the number of permutations with all cycles even,  $o_n$  the number of permutations with all cycles odd, and  $p_n = n!$  the total number of permutations. Let E(x), O(x), and P(x) denote the corresponding exponential generating functions. Prove the following:
  - (a)  $P(t) = (1 t^2)^{-1}$
  - (b)  $E(t) = (1 t^2)^{-1/2}$
  - (c) E(t)O(t) = P(t)
  - (d)  $e_n = o_n$  for all even n.