## Math 301: Homework 1

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due 9 Sept2015

Complete the following problems. Fully justify each response.

- 1. How many words  $(a_1, a_2, \ldots, a_m) \subset [n]^m$  satisfy  $a_1 < a_2 < \cdots < a_m$ ?
- 2. How many integer-valued solutions are there for the following equation, subject to the listed restrictions?

$$x_1 + x_2 + x_3 + x_4 = 152$$

- (a)  $x_1, x_2, x_3, x_4 > 0$
- (b)  $x_1, x_2, x_3 > 0$  and  $x_4 \ge 0$
- (c)  $x_1, x_2, x_3 > 0$  and  $x_4 \ge 0$  and  $x_2 \le 15$
- 3. Let k and n be integers such that  $0 \le k \le n-1$ . Provide a combinatorial proof of the identity

$$\sum_{j=0}^{k} \binom{n}{j} = \sum_{j=0}^{k} \binom{n-1-j}{k-j} 2^{j}.$$

(Note: by a "combinatorial proof," we mean a proof that is based on counting, rather than an algebraic proof.)