Trig integrals:

1. $\int \sin x \; dx = -\cos x + C$
2. $\int \cos x \; dx = \sin x + C$
3. $\int \tan x \; dx = -\ln|\cos x| + C$
4. $\int \sec x \; dx = \ln|\sec x + \tan x| + C$
5. $\int \cot x \; dx = \ln|\sin x| + C$
6. $\int \csc x \; dx = \ln|\csc x - \cot x| + C$
7. $\int \sec^2 x \; dx = \tan x + C$
8. $\int \csc^2 x \; dx = -\cot x + C$
9. $\int \sec x \tan x \; dx = \sec x + C$
10. $\int \csc x \cot x \; dx = -\csc x + C$

Trig derivatives:

1. $\frac{d}{dx}(\sin(x)) = \cos(x)$
2. $\frac{d}{dx}(\cos(x)) = -\sin(x)$
3. $\frac{d}{dx}(\tan(x)) = \sec^2(x)$
4. $\frac{d}{dx}(\csc(x)) = -\csc(x)\cot(x)$
5. $\frac{d}{dx}(\sec(x)) = \sec(x)\tan(x)$
6. $\frac{d}{dx}(\cot(x)) = -\csc^2(x)$

Inverse trig derivatives:

1. $\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$
2. $\frac{d}{dx}(\cos^{-1}(x)) = -\frac{1}{\sqrt{1-x^2}}$
3. $\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$
4. $\frac{d}{dx}(\csc^{-1}(x)) = -\frac{1}{x\sqrt{x^2-1}}$
5. $\frac{d}{dx}(\sec^{-1}(x)) = \frac{1}{x\sqrt{x^2-1}}$
6. $\frac{d}{dx}(\cot^{-1}(x)) = -\frac{1}{1+x^2}$

Trig Identities

1. $\sin^2\theta + \cos^2\theta = 1$
2. $\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$
3. $\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$
4. $\sin(2\theta) = 2\sin\theta\cos\theta$
5. $\cos(2\theta) = \cos^2\theta - \sin^2\theta$
6. $\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$
7. $\sin(\theta/2) = \pm\sqrt{(1-\cos\theta)/2}$
8. $\cos(\theta/2) = \pm\sqrt{(1+\cos\theta)/2}$

Integrals involving $\sqrt{1 \pm x^2}$

1. $\int \frac{1}{\sqrt{1+x^2}} \; dx = \sinh^{-1}(x) + C$
2. $\int \frac{1}{\sqrt{x^2-1}} \; dx = \cosh^{-1}(x) + C$
3. $\int \frac{1}{1-x^2} \; dx = \tanh^{-1}(x) + C$
4. $\int \frac{1}{|x|\sqrt{x^2+1}} \; dx = -\operatorname{csch}^{-1}(x) + C$
5. $\int \frac{1}{x\sqrt{1-x^2}} \; dx = -\operatorname{sech}^{-1}(x) + C$
6. $\int \frac{1}{1-x^2} \; dx = \coth^{-1}(x) + C$
7. $\int \frac{1}{1-x^2} \; dx = \frac{1}{2}\ln\left|\frac{x+1}{x-1}\right| + C$
8. $\int \frac{1}{x^2-1} \; dx = \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| + C$
9. $\int \frac{1}{\sqrt{x^2+1}} \; dx = \ln(x + \sqrt{x^2+1}) + C$
10. $\int \frac{x^2}{\sqrt{1+x^2}} \; dx = \frac{x}{2}\sqrt{1+x^2} - \frac{1}{2}\ln(x+\sqrt{1+x^2}) + C$
11. $\int \frac{1}{x\sqrt{1+x^2}} \; dx = -\ln\left|(\sqrt{x^2+1}+1)/x\right| + C$
12. $\int \frac{1}{x^2\sqrt{1+x^2}} \; dx = -(\sqrt{1+x^2})/x + C$

Area: Let D be a region in the xy-plane. The area of D is given by $A(D) = \iint\limits_D 1 dA$.

Volume: Let E be a solid in $\mathbb{R}^3$. The volume of E is given by $V(E) = \iiint\limits_E 1 dV$.

Average Value: Let $f(x, y)$ be a function defined on a region D. The average value of f over D is

$$\frac{1}{A(D)} \iint_D f(x, y) dA.$$

Surface Area: Let $z = f(x, y)$ define a surface S over a region D. The surface area of S is

$$A(S) = \iint_D \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2 + 1}\, dA.$$

Rectangular/Cylindrical Conversion: A point (x, y, z) in rectangular coordinates may be written as (r, θ, z) in cylindrical coordinates, where

$$r^2 = x^2 + y^2, \quad x = r\cos\theta, \quad y = r\sin\theta, \quad z = z.$$

Given a triple integral, we may write $dV = dx\ dy\ dz = r\ dz\ dr\ d\theta$.

Rectangular/Spherical Conversion: A point (x, y, z) in rectangular coordinates may be written as (ρ, θ, ϕ) in spherical coordinates, where

$$\rho^2 = x^2 + y^2 + z^2, \quad x = \rho\sin\phi\cos\theta, \quad y = \rho\sin\phi\sin\theta, \quad z = \rho\cos\phi.$$

Given a triple integral, we may write $dV = dx\ dy\ dz = \rho^2 \sin\phi\ d\rho\ d\theta\ d\phi$.

Jacobians: Let $x = g(u, v)$, $y = h(u, v)$ be a coordinate change. The Jacobian of this transformation is given by

$$\begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix} = x_u y_v - x_v y_u.$$

If $x = g(u, v, w), y = h(u, v, w), z = k(u, v, w)$ is a coordinate change, then the Jacobian of this transformation is given by

$$\begin{vmatrix} x_u & x_v & x_w \\ y_u & y_v & y_w \\ z_u & z_v & z_w \end{vmatrix} = x_u(y_v z_w - y_w z_v) - x_v(y_u z_w - y_w z_u) + x_w(y_u z_v - y_v z_u).$$

Line Integral: Let C be a curve parameterized by $\mathbf{r}(t) = \langle x(t), y(t)\rangle$ for $a \le t \le b$, and let $f(x, y)$ be a continuous function defined along C. Then we define:

- $\int_C f(x, y) ds = \int_a^b f(x(t), y(t)) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\ dt.$
- $\int_C f(x, y) dx = \int_a^b f(x(t), y(t)) x'(t) dt$
- $\int_C f(x, y) dy = \int_a^b f(x(t), y(t)) y'(t) dt$

Line Integral over Vector Fields: Let C be a curve parameterized by $\mathbf{r}(t) = \langle x(t), y(t)\rangle$ for $a \le t \le b$, and let $\mathbf{F}(x, y) = \langle P(x, y), Q(x, y)$ be a vector field. Then

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_C (P dx + Q dy) = \int_C \mathbf{F}(\mathbf{r}(t)) \cdot \mathbf{r}'(t) dt = \int_C \mathbf{F} \cdot \mathbf{T} ds.$$

Green's Theorem: Let C be a positively oriented, piecewise-smooth, simple closed curve in the plane, and let D be the region bounded by C. If P and Q have continuous partial derivatives in an open region that contains D, then

$$\int_C (P dx + Q dy) = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA.$$