Derivatives and Integrals

Exponential/log:

1. $\frac{d}{dx}\left(b^x\right) = b^x \ln b$
2. $\frac{d}{dx}\left(\log(f(x))\right) = \frac{f'(x)}{f(x)}$

Trig derivatives:

1. $\frac{d}{dx}\left(\sin(x)\right) = \cos(x)$
2. $\frac{d}{dx}\left(\cos(x)\right) = -\sin(x)$
3. $\frac{d}{dx}\left(\tan(x)\right) = \sec^2(x)$
4. $\frac{d}{dx}\left(\csc(x)\right) = -\csc(x)\cot(x)$
5. $\frac{d}{dx}\left(\sec(x)\right) = \sec(x)\tan(x)$
6. $\frac{d}{dx}\left(\cot(x)\right) = -\csc^2(x)$

Inverse trig derivatives:

1. $\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$
2. $\frac{d}{dx}\left(\cos^{-1}(x)\right) = -\frac{1}{\sqrt{1-x^2}}$
3. $\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2}$
4. $\frac{d}{dx}\left(\csc^{-1}(x)\right) = -\frac{1}{x\sqrt{x^2-1}}$
5. $\frac{d}{dx}\left(\sec^{-1}(x)\right) = \frac{1}{x\sqrt{x^2-1}}$
6. $\frac{d}{dx}\left(\cot^{-1}(x)\right) = -\frac{1}{1+x^2}$

Trig Identities

1. $\sin^2\theta + \cos^2\theta = 1$
2. $\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$
3. $\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$
4. $\sin(2\theta) = 2\sin\theta\cos\theta$
5. $\cos(2\theta) = \cos^2\theta - \sin^2\theta$
6. $\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$
7. $\sin(\theta/2) = \pm\sqrt{\frac{1-\cos\theta}{2}}$
8. $\cos(\theta/2) = \pm\sqrt{\frac{1+\cos\theta}{2}}$

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Chain Rule

Suppose that u is a differentiable function of the n variables $x_1, x_2, \ldots, x_n$, and each x_j is a differentiable function of the m variables $t_1, t_2, \ldots, t_m$. Then u is a function of $t_1, \ldots, t_m$, and moreover

$$\frac{\partial u}{\partial t_j} = \frac{\partial u}{\partial x_1}\frac{\partial x_1}{\partial t_j} + \frac{\partial u}{\partial x_2}\frac{\partial x_2}{\partial t_j} + \cdots + \frac{\partial u}{\partial x_n}\frac{\partial x_n}{\partial t_j}.$$

Second Derivative Test

Suppose that the second partial derivatives of f are continuous on a disk with center (a, b), and suppose that $f_x(a, b) = 0$ and $f_y(a, b) = 0$. Let

$$D = D(a, b) = f_{xx}(a, b) f_{yy}(a, b) - \left[f_{xy}(a, b)\right]^2.$$

1. If $D > 0$ and $f_{xx}(a, b) > 0$ then $f(a, b)$ is a local minimum.
2. If $D > 0$ and $f_{xx}(a, b) < 0$ then $f(a, b)$ is a local maximum.
3. If $D < 0$ then $f(a, b)$ is not a local minimum or maximum.