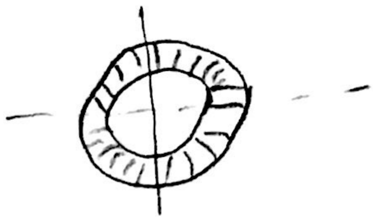
2) Volume below surface $z = f(x,y) = e^{-x^2-y^2}$

$\Rightarrow$ We need to evaluate $\iint_D f(x,y)\, dA$ D = Annulus



We will parameterize our domain using polar coordinates as the graphs are just circles

$$D = \begin{cases} 0 \leq \theta \leq 2\pi \\ 1 \leq r \leq 2 \end{cases}$$

Thus our integral transforms to:

$$\int_0^{2\pi}\int_1^2 e^{-r^2}\, r\, dr\, d\theta$$

$u = r^2$

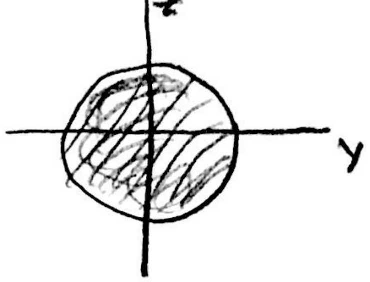
$du = 2r\,dr$

$$= \frac{1}{2}\int_0^{2\pi}\int_1^4 e^{-u}\, du\, d\theta = \pi\left[-e^{-u}\right]_1^4 = \boxed{\pi\left[e^{-1} - e^{-4}\right]}$$

4) Just reading off the bounds

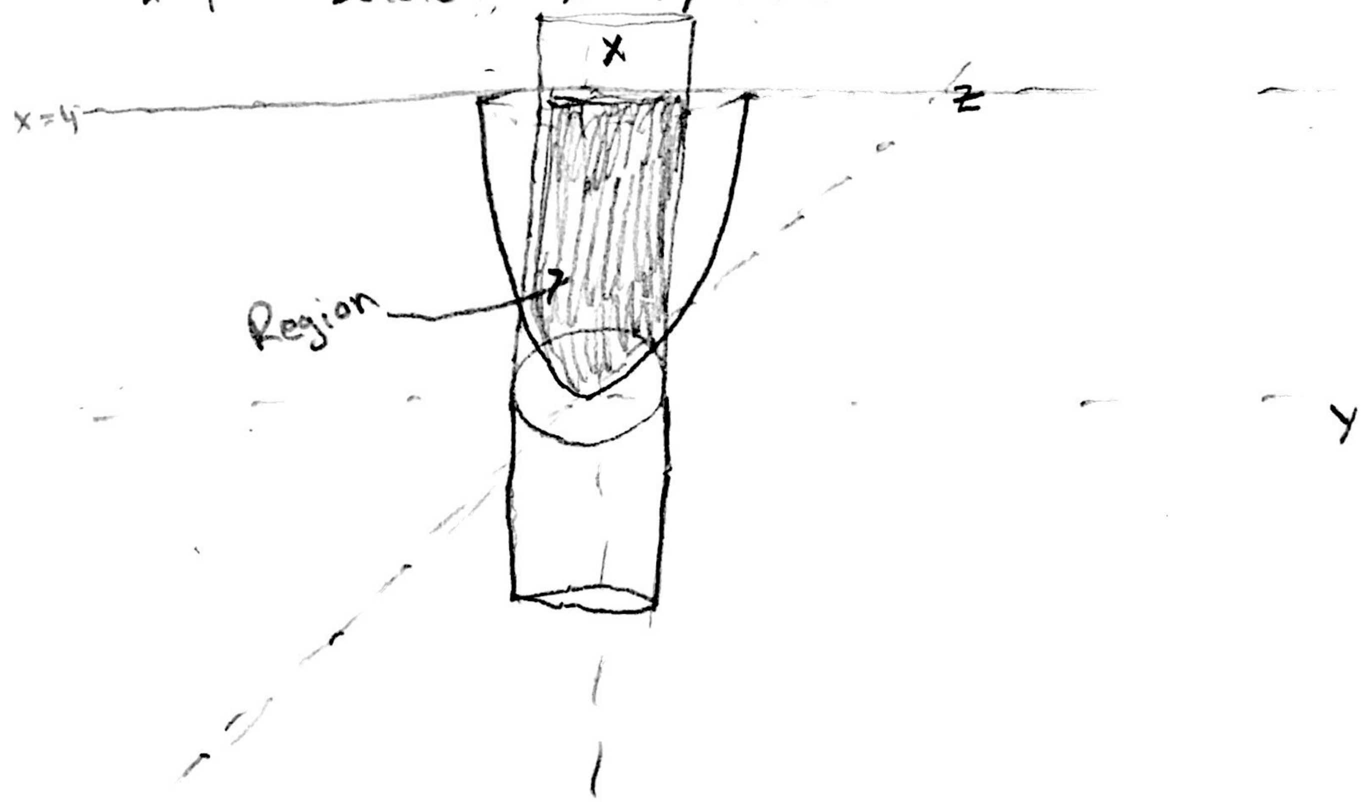
$$\Rightarrow D = \begin{cases} 4y^2+4z^2 \le x \le 4 \\ -\sqrt{1-y^2} \le z \le \sqrt{1-y^2} \\ -1 \le y \le 1 \end{cases}$$

So in the yz plane our region is $y^2+z^2=1$ $-1 \le y \le 1$



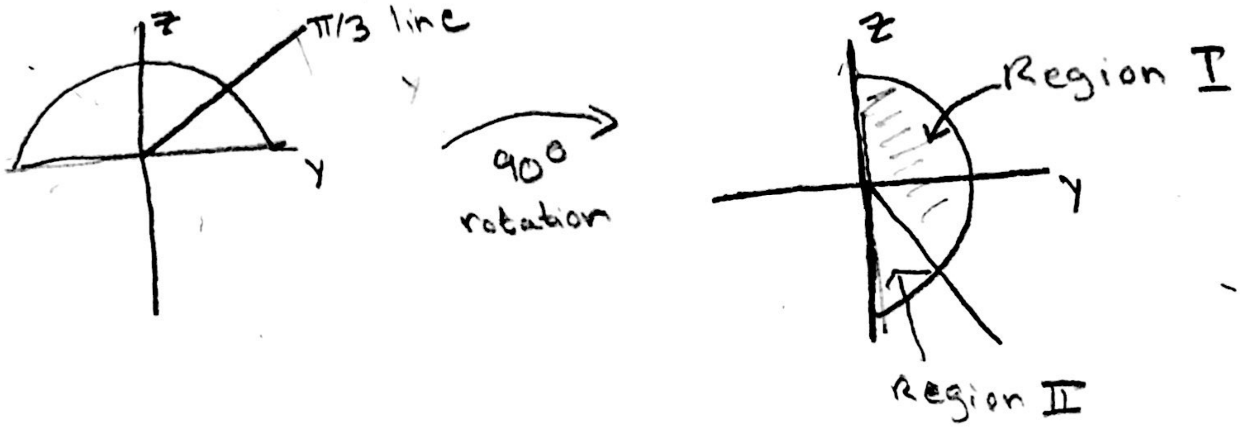
Note that $x = 4y^2+4z^2$ is paraboloid

Thus this surface is bounded by the cylinder $y^2+z^2=1$, plane $x=4$, & paraboloid $x=4y^2+4z^2$



6) By symmetry of the hemisphere we can rotate the coordinate system 90° to make it simple to visualize and parameterize.

Thus our yz projection goes from



Now the two regions are easy to parameterize in terms of ϕ.

Region S: $\begin{cases} 0 \le \rho \le 4 \\ -\frac{\pi}{2} \le \theta \le \frac{\pi}{2} \end{cases}$ ← as we only have hemisphere

$0 \le \phi \le \frac{2\pi}{3}$ for Region I

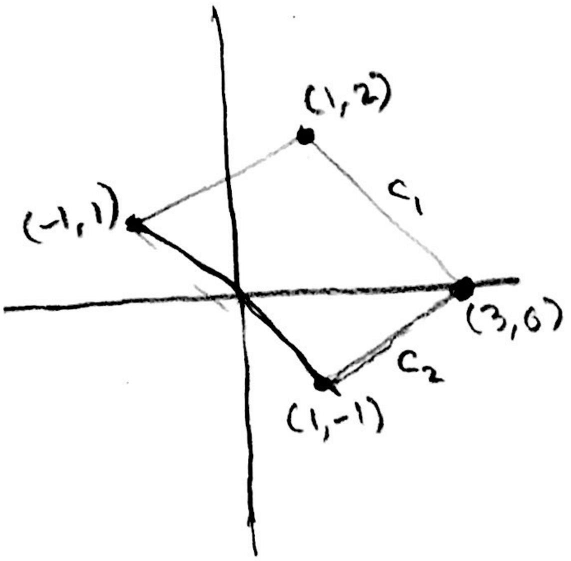
$\frac{2\pi}{3} \le \phi \le \pi$ for Region II.

Thus, Vol(Region I) $= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{\frac{2\pi}{3}} \int_0^4 \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$

$$= \boxed{\pi \left[\frac{1}{3}(4)^3\right]\left[\cos(0) - \cos\left(\frac{2\pi}{3}\right)\right]}$$

Similarly Vol(Region II) $= \pi\left[\frac{1}{3}(4)^3\right]\left[\cos\left(\frac{2\pi}{3}\right) - \cos(\pi)\right]$

8) Drawing this out we see the rectangle is tilted



So let us do a change of variables to make the Domain Easier to integrate.

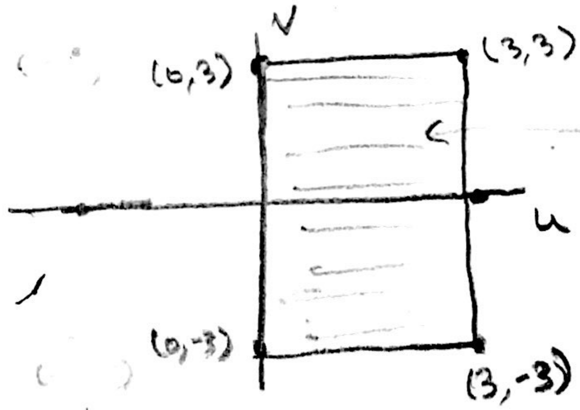
Note that $C_1: y = -x + 3$

$C_2: y = \frac{1}{2}x - \frac{3}{2}$

We want to find u, v s.t these two lines map to constants. Which we can do by setting $u = x + y$ (Sets C_1 to 3)

$v = 2y - x$ (sets C_2 to -3)

Thus in the u, v plane we get



And note that $u + v = 3y \Rightarrow y = \frac{1}{3}(u+v)$

$\Rightarrow x = u - \frac{1}{3}(u+v) = \frac{2}{3}u - \frac{1}{3}v$

$$\Rightarrow J = \begin{vmatrix} \frac{2}{3} & -\frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} \end{vmatrix} \Rightarrow |\det J| = \frac{2}{9} - \frac{1}{9} = \frac{1}{9}$$

Thus our integral becomes

$$I = \frac{1}{9} \iint\limits_{\substack{[0,3] \times [-3,3] \\ u \qquad\quad v}} \frac{v}{u} \, dA$$

Note that this function is discontinuous at the origin.
So we need to treat it like an improper integral and
cutout $\underset{u}{[0,\varepsilon)} \times \underset{v}{(-\varepsilon,\varepsilon)}$ and use Fubini on The rest as it is continuous there.

$$\text{So } I = \lim_{\varepsilon \to 0} \frac{1}{9} \left[\int_{\varepsilon}^{3} \left[\int_{\varepsilon}^{3} \frac{v}{u} \, dv + \int_{-3}^{-\varepsilon} \frac{v}{u} \, dv \right] du \right]$$

$$= \lim_{\varepsilon \to 0} \frac{1}{9} \left[\int_{\varepsilon}^{3} \underbrace{\left[\left[\frac{v^2}{2u} \right]_{\varepsilon}^{3} + \left[\frac{v^2}{2u} \right]_{-3}^{-\varepsilon} \right]}_{= 0} du \right]$$

$$\Rightarrow \text{Integral} = 0$$

16) Note that $\vec{F}$ is conservative

as if $f(x,y) = \frac{x^2}{2} + 2x + \frac{y^2}{2}$

Then $\nabla f = \vec{F} = \langle x+2, y \rangle$

Thus, we can use the fundamental Thm of Line integrals.

$$\Rightarrow \int_C \vec{F} \cdot d\vec{r} = f(r(b)) - f(r(a))$$

$r(0) = (0,0) \quad r(2\pi) = (0, 2\pi)$

$$\Rightarrow \int_C \vec{F} \cdot d\vec{r} = \frac{1}{2}(2\pi)^2$$

12) Note that F has no singularities and is smooth.

And circle satisfies the conditions to use Green's Thm.

Thus $\frac{\partial P}{\partial y} = 1$ $\quad \frac{\partial Q}{\partial x} = 3$

$$\Rightarrow \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 2$$

Thus by Green's Thm:

$$\int_C \vec{F} \cdot d\vec{r} = 2\iint_D dA = 2 \cdot \text{Area}(\text{Circle})$$

$= 4\pi r^2$ In our case circle has radius 2

$\Rightarrow \quad = 16\pi$