Math 228: Homework 6

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due 4 Nov 2016

Complete the following problems. Fully justify each response. You need only turn in those problems marked with a (*).

1. (*) Let G be a graph, and define $\overline{G}$ to be the graph having $V(\overline{G}) = V(G)$, and $E(\overline{G}) = E(G)^c$, that is, $\overline{G}$ has exactly the edges that G doesn't ($\overline{G}$ is called the complement of G).

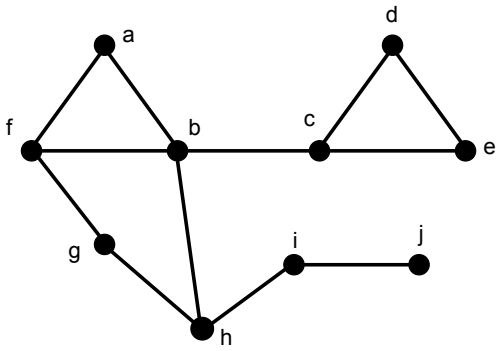
   Prove that at least one of G or $\overline{G}$ is connected.

2. Complete exercise 7.3.3 on page 139.

3. (*) Complete exercise 7.3.12 on page 140.

4. (*) Recall that a cut-edge of a graph G is an edge that, when removed, increases the number of connected components of G. So, if G is connected, for example, a cut-edge is an edge whose removal leaves a disconnected graph.

   (a) Which edges of the graph shown are cut edges?



   (b) Let G be a connected graph. Prove that $e \in E$ is a cut-edge of G if and only if e is not contained in a cycle of G.

5. (*) Let G be a graph. Prove that G is a tree if and only if every two vertices $u, v \in V(G)$ are connected by a unique path.

6. (*) A tree T is called a *double star* if it contains exactly two vertices that are not leaves.

   (a) Draw all the double stars on 7 vertices.

(b) How many unlabeled double stars are there on n vertices?

7. Complete exercise 8.5.12 on page 156.