## Math 228: Homework 8

Mary Radcliffe

due 21 Nov2016

Complete the following problems. Fully justify each response. You need only turn in those problems marked with a (\*).

- 1. (\*) Suppose we have n lines in the plane in general position, and then k other lines that are all parallel to each other, but not parallel to any of the original n lines. How many regions will the plane be divided into if all n + k lines are drawn? Prove that your answer is correct.
- 2. (\*) Complete problem 11.3.2 on page 186.
- 3. Complete problem 11.3.7 on page 187.
- 4. (\*) Complete problem 11.3.8 on page 187.
- 5. (\*) Let P be a convex hexagon. Suppose you triangulate P by dividing P into some triangles using diagonals (see figure). In how many ways can this be done?



Figure 1: A triangulation of a hexagon.

- 6. Let P be a convex n-gon. Suppose you triangulate P by dividing P into some triangles using diagonals (see figure). In how many ways can this be done?
- 7. (\*) For any n, define H(n) to be the smallest number such that any set of H(n) points in general position in the plane contains a convex n-gon, and moreover that convex n-gon contains no other points from the set in its interior. (This is sometimes called an *empty* polygon.)

Prove that H(3) = 3 and H(4) = 5.

(Note: It's also known that H(5) = 10, and that H(6) is finite. H(7) is known to be infinite; that is, you cannot guarantee an empty convex 7-gon no matter how many points you include. But H(6) is still unknown.)