Math 228: Homework 9

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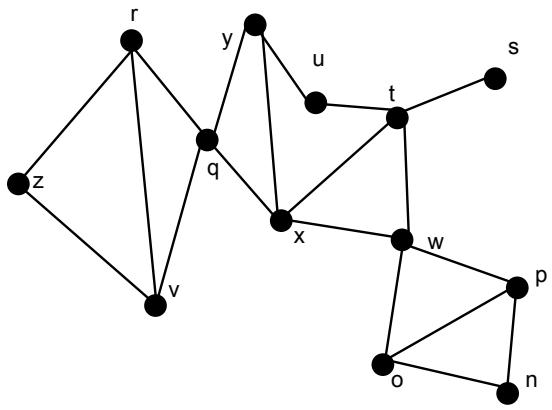
Complete the following problems. Fully justify each response. You need only turn in those problems marked with a (*).

1. (*) Let G be a planar graph on n vertices. Prove that the number of edges in G is at most $3n - 6$.

2. Let G be a graph. Define a *block* of G to be a subgraph of G that is 2-connected, and has no cut-vertices; that is, a block is a subgraph such that removing any edge OR any vertex yields a connected graph.

   (a) Identify the blocks in the following graph by the vertices contained in each block. A vertex may appear in more than one block.



   (b) Show that a graph is planar if and only if each of its blocks is planar. You may want to use induction on the number of blocks.

3. (*) Suppose that G is a 2-connected graph such that every vertex in G has degree at least 3. Show that there exists an edge $uv \in E(G)$ such that $G \backslash \{uv\}$ is also 2-connected.

4. (*) Complete problem 12.3.1 on page 195.

5. Complete problem 12.3.5 on page 195.

6. Complete problem 13.3.1 on page 204.

7. (*) Complete problem 13.3.4 on page 204.

8. An *independent set* in a graph G is a set of vertices in G that contain no edges of G. Show that coloring a graph with k colors is the same as finding a partition of $V(G)$ into k independent sets.

9. (*) Complete problem 13.4.4 on page 210. The graph constructed in this way is called the *line graph* of K_5, and certainly every graph could yield a line graph formed in the same way.

10. (*) Complete problem 13.4.5 on page 210.