# Math 127 Homework 

Mary Radcliffe

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Complete the following problems. Fully justify each response. You need only turn in those problems marked with a (*).

1. (*) Let $a, b, c$ be integers. Prove that if $a$ divides $b$ and $b$ divides $c$, then $a$ also divides $c$.
2. $\left(^{*}\right)$ Let $n, m$ be integers. Prove that $n+m$ is odd if and only if one of $n, m$ is even and the other is odd.
3. $\left(^{*}\right)$ Find all real solutions $x$ to the equation $\sqrt{x+1}+\sqrt{x-3}=4$. Prove that your answer is correct.
4. Follow the method of Example 9 to prove that if $p$ is a positive prime integer, then $\sqrt{p}$ is irrational.
5. Let $x$ be a rational number and $y$ an irrational number. Prove that $x+y$ is irrational using the method of contradiction.
6. (*) Complete the proof of Example 12: that is, show that if $a, b, c$ are all odd integers, and $k$ and $\ell$ are both integers, and $a k^{2}+b k \ell+c \ell^{2}=0$, then $k$ and $\ell$ are both even.
7. Using Example 6 as a model, show that if $n$ is a positive integer, then $n$ is divisible by 9 if and only if the sum of the base 10 digits of $n$ is divisible by 9 .
8. (*) Use the method of proof by contradiction to prove that there are infinitely many prime numbers. You may wish to use the result from Example 11 to help you.
