Math 127 Homework

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Due 31 January 2019

Complete the following problems. Fully justify each response. You need only turn in those problems marked with a (*).

- 1. (*) Let a, b, c be integers. Prove that if a divides b and b divides c, then a also divides c.
- 2. (*) Let n, m be integers. Prove that n + m is odd if and only if one of n, m is even and the other is odd.
- 3. (*) Find all real solutions x to the equation $\sqrt{x+1} + \sqrt{x-3} = 4$. Prove that your answer is correct.
- 4. Follow the method of Example 9 to prove that if p is a positive prime integer, then \sqrt{p} is irrational.
- 5. Let x be a rational number and y an irrational number. Prove that x + y is irrational using the method of contradiction.
- 6. (*) Complete the proof of Example 12: that is, show that if a, b, c are all odd integers, and k and ℓ are both integers, and $ak^2 + bk\ell + c\ell^2 = 0$, then k and ℓ are both even.
- 7. Using Example 6 as a model, show that if n is a positive integer, then n is divisible by 9 if and only if the sum of the base 10 digits of n is divisible by 9.
- 8. (*) Use the method of proof by contradiction to prove that there are infinitely many prime numbers. You may wish to use the result from Example 11 to help you.