21-127 HW1 SOL'NS

① PROOF BY TRUTH TABLE

P	Q	$\neg(P\wedge Q)$	$\neg P\vee\neg Q$	$\neg(P\vee Q)$	$\neg P\wedge\neg Q$
T	T	F	F	F	F
T	F	T	T	F	F
F	T	T	T	F	F
F	F	T	T	T	T

✓ ✓

$\Rightarrow$

$$\neg(P\wedge Q) \equiv \neg P \vee \neg Q$$
$$\neg(P \vee Q) \equiv \neg P \wedge \neg Q$$

---

② $P \leftrightarrow Q \equiv (P\to Q)\wedge(Q\to P)$

P	Q	$P\to Q$	$Q\to P$	$(P\to Q)\wedge(Q\to P)$	$P\leftrightarrow Q$
T	T	T	T	T	T
T	F	T	F	F	F
F	T	F	T	F	F
F	F	T	T	T	T

✓

---

③ A: "x is a positive integer" B: "$x=2$"

C: "x is even" D: "$x>2$" E: "x is prime"

F: "x is composite".

$$A \to (B \vee (C\wedge D) \vee (\neg C\wedge E) \vee (\neg C\wedge F)$$

④

A: $P \wedge Q$ B: $P \wedge \neg R$ C: $Q \wedge R$

D: $\neg P \vee R$

P	Q	R	A	B	C	D	$\neg(A \vee B \vee C)$	$\neg C \wedge D$
T	T	T	T	F	T	T	F	F
T	T	F	T	T	F	F	F	F
T	F	T	F	F	F	T	T	T
T	F	F	F	T	F	F	F	F
F	T	T	F	F	T	T	F	F
F	T	F	F	F	F	T	T	T
F	F	T	F	F	F	T	T	T
F	F	F	F	F	F	T	T	T

⑤

$$(P \to Q) \vee \neg Q \equiv T$$

P	Q	$P \to Q$	$(P \to Q) \vee \neg Q$
T	T	T	T
T	F	T	T
F	T	F	T
F	F	T	T

✓

⑥ Let $x, y \in \mathbb{Q}$. Then $x = \frac{m}{n}$, $y = \frac{m'}{n'}$,

$m, n, m', n' \in \mathbb{Z}$.

then

$xy = \frac{mm'}{nn'}$, $mm' \in \mathbb{Z}$, $nn' \in \mathbb{Z} \Rightarrow xy \in \mathbb{Q}$.

$\frac{x}{y} = \frac{mn'}{nm'}$, $mn' \in \mathbb{Z}$, $nm' \in \mathbb{Z} \Rightarrow \frac{x}{y} \in \mathbb{Q}$.

$x - y = \frac{mn' - m'n}{mn'}$, $mn' - m'n \in \mathbb{Z}$, $nn' \in \mathbb{Z} \Rightarrow x - y \in \mathbb{Q}$.

(7) Let $d, a, b, u, v \in \mathbb{N}$. $d \mid a \Rightarrow a = dk$ for some $k \in \mathbb{N}$.

$d \mid b \Rightarrow$ ~~and~~ $b = d\ell$ for some $\ell \in \mathbb{N}$.

Then $au + bv = (dk)u + (d\ell)v =$

$d(ku + \ell v)$. Since $ku + \ell v \in \mathbb{N}$,

we conclude $d \mid (au+bv)$.

(8) x solves $ax^2 + bx + c = 0 \leftrightarrow x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

"$\rightarrow$". Let $ax^2 + bx + c = 0$. Then by algebra,

$$x^2 + \frac{b}{a}x = \frac{-c}{a}$$

$$\leftrightarrow x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2 - 4ac}{4a^2}$$

$$\leftrightarrow \left(x + \frac{b}{2a}\right)^2 = \left(\frac{b^2-4ac}{4a^2}\right)$$

$$\rightarrow x + \frac{b}{2a} = \pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$\rightarrow x = \cancel{-\frac{b}{2a}} \; \frac{-b \pm \sqrt{b^2-4ac}}{2a}.$$

"$\leftarrow$". Let $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$, then

$$ax^2 + bx + c = a\left(\frac{-b \pm \sqrt{b^2-4ac}}{2a}\right)^2 + b\left(\frac{-b \pm \sqrt{b^2-4ac}}{2a}\right) + c$$

$$= \frac{b^2 \mp 2b\sqrt{b^2-4ac} + b^2 - 4ac}{4a} + \frac{-b^2 \pm \sqrt{b^2-4ac}}{2a} + c$$

$$= -c + c = 0.$$