

# Math 301 Homework

Mary Radcliffe

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Complete the following problems. Fully justify each response.

1. Let  $a, b, c$  be integers. Suppose that  $c|a$  and  $c|b$ . Prove that  $c|(a + b)$ .
2. Let  $a$  and  $b$  be irrational numbers, and let  $x$  be a rational number. Determine if the following are irrational, rational, or whether it cannot be determined. Prove that your answer is correct.
  - (a)  $a + b$
  - (b)  $a + x$
  - (c)  $ax$
  - (d)  $ab$
3. Let  $a, b, r$  be integers, and suppose that  $a$  leaves a remainder of  $r$  when divided by  $b$ . Prove that  $a$  also leaves a remainder of  $r$  when divided by  $-b$ .
4. Let  $a, b \in \mathbb{R}$  and let  $p(x) = x^2 + ax + b$ . The value  $\Delta = a^2 - 4b$  is called the discriminant of  $p$ . Prove that  $p$  has no real roots if  $\Delta < 0$ , one real root if  $\Delta = 0$ , and two real roots if  $\Delta > 0$ .
5. Let  $p(x)$  be a polynomial with coefficients in  $\mathbb{R}$ . Suppose that  $\alpha$  is a root of  $p(x)$ . Prove that  $\alpha$  is also a root of  $(x - a)p(x)$  for all  $a \in \mathbb{R}$ .
6. Identify which of the following are propositions, and discuss what might be required to prove them if they are.
  - (a) This is the tallest sheep in the world.
  - (b) My dog Rufus.
  - (c) The square of every even integer is also an even integer.
  - (d) A kitty is orange.
  - (e) This kitty is orange.
  - (f) Go to the store.