Math 301 Homework

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Complete the following problems. Fully justify each response.

1. Prove that for any $n \in \mathbb{N}$, $4^n + 6n - 1$ is divisible by 9.
2. Prove that for any $n \in \mathbb{N}$,
$$\prod_{i=0}^{n-1}(2i+1) = \frac{(2n)!}{2^n n!}.$$
3. Prove the following theorem in 2 ways, first by induction on n, and second by the Binomial Theorem.

   For any $n \in \mathbb{N}$, we have $\displaystyle\sum_{i=0}^{n}\binom{n}{i}(-1)^i = 0$.
4. Suppose we play a game where there are 2 players. The game is as follows: first, make two nonempty piles of pennies. On each player's turn, they may remove as many pennies as they like from one of the piles (but not 0). The player who removes the last penny wins the game.

   Prove that if the two piles initially contain the exact same number of pennies, then the second player can always win the game. Prove that if the two piles initially contain different numbers of pennies, then the first player can always win the game.
5. Write truth tables for $p \wedge (q \wedge r)$ and $p \wedge (q \vee r)$. Where do they differ? Why?
6. Prove part (b) of Theorem 2.1.14 (De Morgan's Law).