

Math 127 Homework

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Complete the following problems. Fully justify each response.

1. Let X, Y, Z be sets, with $X, Y \subseteq Z$. Prove that

$$[(Z \setminus X) \cap (Z \setminus Y)] \cup (X \setminus Y) = Z \setminus Y.$$

2. Let X be a set. Prove that $X \times \emptyset = \emptyset$.
3. Let X, Y, Z be sets. Is it true that $X \times (Y \times Z) = (X \times Y) \times Z$? Explain your answer with a proof or a counterexample.
4. For each of the following subsets G of $X \times Y$, determine if the subset represents the graph of a function from $X \rightarrow Y$. If so, specify the function.
- (a) $X = \mathbb{R}, Y = \mathbb{R}, G = \{(x, x+1) \mid x \in \mathbb{R}\}$.
 - (b) $X = \mathbb{R}, Y = \mathbb{R}, G = \{(x^2, x) \mid x \in \mathbb{R}\}$.
 - (c) $X = \mathbb{R}^+, Y = \mathbb{R}^+, G = \{(x^2, x) \mid x \in \mathbb{R}^+\}$.
 - (d) $X = \mathbb{Q}, Y = \mathbb{Q}, G = \{(x, y) \mid x, y \in \mathbb{Q} \text{ and } xy = 1\}$.
5. Which of the following function specifications are well-defined? If one is not well-defined, determine a modification to the specification that would rectify the issue.
- (a) $g : \mathbb{Q} \rightarrow \mathbb{Q}$ defined by $g(x)(x+1) = 2$.
 - (b) $f : \mathbb{Q} \rightarrow \mathbb{R}$ defined by $f(x)(x+\pi) = 1$.
 - (c) $h : \mathbb{R} \rightarrow \mathbb{R}$ defined by $h(x) = \sqrt{x}$.
 - (d) $\ell : \mathbb{C} \rightarrow \mathbb{C}$ defined by $\ell(x) = \sqrt{x}$.

6. Let $f, g, h, \ell : \mathbb{R} \rightarrow \mathbb{R}$ be functions with the following specifications:

$$f(x) = x + 2; \quad g(x) = x^2; \quad h(x) = \frac{1}{x^2 + 1}; \quad \ell(x) = -x.$$

Write a specification, via a single equation, for each of the following:

- (a) $f \circ g$.
- (b) $g \circ f$.
- (c) $f \circ (g \circ (h \circ \ell))$.
- (d) $(f \circ g) \circ (h \circ \ell)$.