Math 301 Homework

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Complete the following problems. Fully justify each response.

1. Let p and q be propositional variables. Prove that $p \iff q$ is logically equivalent to $\neg\left(((\neg p) \wedge q) \vee (p \wedge (\neg q))\right)$.
2. Suppose it is known that every continuous function on a closed interval $[a, b]$ in $\mathbb{R}$ has a maximum value. Now, suppose I have a function f, which I tell you has a maximum value on the interval $[a, b]$. What can you conclude about the continuity of f? Use the language of propositional logic to explain your answer.
3. Let $p(x, y)$ be the statement that $xy = 1$. Suppose that x is a member of the positive integers $\{1, 2, 3, \dots\}$ and y is a member of the rationals. Consider the following two statements:
   (a) $\forall x, \exists y, p(x, y)$
   (b) $\exists y, \forall x, p(x, y)$

   How do these statements differ? Are either of them true?
4. For each of the following statements about sets, determine if the statement is true or false. If true, prove the statement. If false, explain why.
   (a) For any set X, $\emptyset \in X$.
   (b) For any set X, $\emptyset \subseteq X$.
   (c) $\{x \in \mathbb{Z} \mid x \geq 0\} = \mathbb{N}$.
   (d) $\emptyset \subseteq \mathcal{P}(\emptyset)$.
   (e) $\emptyset \in \mathcal{P}(\emptyset)$.
5. Prove that $X \subseteq Y$ if and only if $X \cap Y = X$.
6. Prove that $X \cup (Y \cap Z) = (X \cup Y) \cap (X \cup Z)$.