Math 301 Homework

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Complete the following problems. Fully justify each response.

1. Let n be a natural number. Prove that n is divisible by 3 if and only if the sum of the base-10 digits of n is divisible by 3.

2. Let $x \in \mathbb{R}$. Prove that if $x \neq 0$, then $x^2 > 0$.

3. Let x be an irrational number. Prove that $\frac{1}{x}$ and $-x$ are both irrational.

4. Prove that for any real number $x \in \mathbb{R}$, there exists a real number $y \in \mathbb{R}$ such that $x + y \in \mathbb{Z}$.

5. Let $p(n)$ be the statement "$2n + 1$ is divisible by 3," for $n \in \mathbb{N}$. Prove that for any $n \in \mathbb{N}$, $p(3n + 1)$ is true, and $p(3n)$ and $p(3n + 2)$ are false. (Note: this is Exercise 1.3.9 on page 50. There is some further discussion there about the statement $p(n)$.)

6. Explain, in your own words, the Principle of Weak Induction (Thm. 1.3.10). Without writing a formal proof, discuss why the theorem is true.