

Math 127 Homework

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Complete the following problems. Fully justify each response.

1. A number is called *algebraic* if it is the root of a polynomial $p(x) = a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0$, where each $a_i \in \mathbb{Z}$. Let \mathcal{A} denote the set of algebraic numbers.
 - (a) Prove that $\mathbb{Q} \subseteq \mathcal{A}$.
 - (b) Prove that the set of all algebraic numbers is countably infinite.
(Hint: First consider the possible roots of polynomials of degree k . Then use a union argument).
2.
 - (a) Let X be any set. Prove, using Cantor's Diagonalization Argument, that $|\mathcal{P}(X)| > |X|$.
 - (b) Prove that $\mathcal{P}(X)$ is either finite or uncountably infinite.
3. Let $f : X \rightarrow Y$ be a function. Define a relation R on X by $x_1Rx_2 \Leftrightarrow f(x_1) = f(x_2)$. Is this an equivalence relation? If so, prove it. If not, explain why not.