# Math 127 Homework

Mary Radcliffe

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Complete the following problems. Fully justify each response.

1. Let $(X, 0, 1, +, \cdot)$ be a field, where $X$ is a finite set. Prove that there is no ordering $\leq$ on $X$ under which $X$ is an ordered field.
2. Let $(X, 0, 1, +, \cdot, \leq)$ be an ordered field. Prove each of the following basic ordered field properties, from axioms.
   (a) For all $x \in X$, $x^2 > 0$.
   (b) For all $w, x, y, z \in X$, if $w \leq x$ and $y \leq z$, then $w + y \leq x + z$.
   (c) For all $x, y, z \in X$, if $x \geq 0$ and $y \leq z$, then $xy \leq xz$.
   (d) For all $x, y, z \in X$, if $x \leq 0$ and $y \leq z$, then $xy \geq xz$.
3. Let $\vec{x}, \vec{y}, \vec{z} \in \mathbb{R}^n$. Prove that $\|\vec{x} - \vec{z}\| \leq \|\vec{x} - \vec{y}\| + \|\vec{y} - \vec{z}\|$.
4. Let $x_n = \frac{n+2}{n+1}$. Prove that $x_n$ converges to 1.
5. Let $(x_n)$ and $(y_n)$ be sequences of real numbers, with $(x_n) \to a$ and $(y_n) \to b$. Let $z_n = x_n y_n$ for all $n \in \mathbb{N}$. Prove that $(z_n) \to ab$.
6. Prove that if $(x_n)$ is a monotonically decreasing sequence, having a lower bound, then $(x_n)$ converges.