

Math 127 Homework

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Complete the following problems. Fully justify each response.

1. Let X be a set, and let \sim_1 and \sim_2 be two equivalence relations on X . Define a relation \sim by $x \sim y$ if and only if $x \sim_1 y \wedge x \sim_2 y$. Prove that this is an equivalence relation. Describe the equivalence classes $[x]_\sim$ in terms of $[x]_{\sim_1}$ and $[x]_{\sim_2}$.
2. Let P be a poset with element set \mathbb{N} , and order \preceq , where $a \preceq b$ if and only if $a|b$. Formally prove that this is a poset.
3. Let P be a poset with partial order \leq . We define an element m of P to be *minimal* if there does not exist any $x \in P$ with $x < m$.
 - (a) How is a minimal element different from a minimum element (as defined in class)?
 - (b) Give an example of a poset that has at least one minimal element, but no minimum element.
 - (c) Prove that if a poset has a minimum element, then it has only one minimal element.

(Note: This entire problem could be repeated verbatim for a maximal element as well)

4. Let X be a set, and let P be the poset whose elements are $\mathcal{P}(X)$, and $A \leq B$ if and only if $A \subseteq B$.

Let $\{A_1, A_2, \dots, A_n\}$ be elements of P . Prove that $\bigwedge_{i=1}^n A_i = \bigcap_{i=1}^n A_i$, and

$$\bigvee_{i=1}^n A_i = \bigcup_{i=1}^n A_i.$$

5. Let (X, \preceq) be a lattice, and let $x, y \in X$. Prove the following:
 - (a) $x \vee y = y \vee x$
 - (b) $x \wedge y = y \wedge x$
 - (c) $x \vee (x \wedge y) = x$
 - (d) $x \wedge (x \vee y) = x$
6. Give an example of a lattice that is not distributive.