Math 101 Homework

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Complete the following problems. Fully justify each response.

1. Suppose M is a graph[1], that has the following condition:

$$\text{if } a \not\sim b \text{ and } b \not\sim c, \text{ then } a \not\sim c$$

   (where $\not\sim$ indicates vertices are not adjacent and $\sim$ indicates vertices are adjacent).

   Prove that the vertex set of M can be divided into subsets $\mathcal{S} = \{S_1, S_2, S_3, \dots\}$ (this list could be finite in length) in such a way that if $u, v \in S_i$ (the same i), then $u \not\sim v$, but if $u \in S_i$ and $v \in S_j$ (with $i \neq j$), then $u \sim v$.

2. Mess around with triangle-free, 3-chromatic unit graphs. Try to build some that don't contain any 5- or 7-cycles (without just being larger odd cycles).

3. Suppose that we lived in a different universe, and Erdős' 1975 conjecture that every triangle-free unit graph is 3-chromatic was proven true instead of false. How do you think that could help mathematicians solve the Hadwiger-Nelson problem? What kind of approaches do you think would made sense if Erdős' conjecture were true?

[1] Here you can assume that M has at most countably many vertices, so that we can list the partition sets in $\mathcal{S}$. In general, this statement is still true when M has more than countably many vertices, but in that case you may not get countably many independence sets in the partition.