

**Math 21-121**  
**Extras for Homework 5**  
WE SHALL REFER IT AS “Extras 5”

1. Express the function  $\sec^{-1}(\sin(x))$  (typo: skip this part) and  $\cot(\sec^{-1}(x))$  as an algebraic function of  $x$ . In particular, express it in a form of a function that does not involve any trigonometric function in it.
2. We know that there is no agreeable range for the function  $f(x) = \sec^{-1}(x)$ . Write your favorite range of the function  $f(x) = \sec^{-1}(x)$  and sketch the graph of the same function keeping that range in mind.
3. Determine the exact value of a)  $\tan^{-1}(\tan[11\pi/4])$     b)  $\cos^{-1}(\sec[7\pi/6])$ .
4. Differentiate the function  $f(x) = \frac{x}{\sqrt{c^2-x^2}} - \sin^{-1}(\frac{x}{c})$ ,  $c > 0$ .
5. A person walking along a straight path at the rate of 6 feet per second is followed by a spotlight that is located 30 feet from the path. How fast is the spotlight turning at the instant the person is 50 feet past the point on the path that is closest to the spotlight?
6. Sketch the graph of  $\sinh(x)$  and  $\cosh(x)$ .
7. Find the numerical value of  $\cosh(\ln 3)$  and  $\sinh(\ln 2)$ .
8. Prove the identity  $(\cosh x + \sinh x)^6 = \cosh 6x + \sinh 6x$ .
9. Find the derivative of the following:
  - (a)  $y = \ln |1 - \sinh ax|$ .
  - (b)  $y = x \cosh x$ .
  - (c)  $y = \tan^{-1}(\sinh x)$ .
  - (d)  $y = \tanh^{-1} \sqrt{x}$
10. Evaluate the limit  $\lim_{x \rightarrow \infty} \frac{\sinh x}{e^x}$ . (DO NOT USE L'Hospital rule)
11. If  $\operatorname{csch} x = \frac{4}{3}$ , find the values of the other hyperbolic functions at  $x$ .
12. Determine  $A$ ,  $B$ , and  $c$  so that  $y = A \cosh cx + B \sinh cx$  satisfies the conditions  $4y'' - y = 0$ ,  $y(0) = 1$ ,  $y'(0) = 2$ . Take  $c > 0$ .
13. At what point of the curve  $y = \cosh x$  does the tangent have slope 1?
14. Show that  $\tanh^{-1} x = \frac{1}{2} \ln(\frac{1+x}{1-x})$ , where  $-1 < x < 1$ .