Problem set on Vector Functions MM

1. Find the domain of **f**(t) = sin*t* **i** + ln(2 - t)**j** + $\sqrt{\boldsymbol{t+1}}$ **k.**

   (Ans. [-1, 2))

2. Given $\boldsymbol{f}(t) = t^2\mathbf{i} + \frac{t}{1+t}\mathbf{j} + \frac{1-cost}{2t}\mathbf{k}$, find $\lim_{t\to 0} \boldsymbol{f}(\mathrm{t})$.

   (Ans. =vector 0)

3. Find the derivative of the following functions wherever it exists.

   - $\boldsymbol{f}(t) = t^2\mathbf{i} + \frac{t}{1+t}\mathbf{j} + \frac{1-cost}{2t}\mathbf{k}$

     (Ans. $\boldsymbol{f'(t) = 2t\, i - \frac{1}{(1+t)^2} j + \frac{2t \sin t - 2(1-\cos t)}{4t^2}\, k}$ **)**

   - $\boldsymbol{g}(t) = cos(4t)\mathbf{i} + \ln(2-t)\mathbf{j} + \mathbf{k}$

     (Ans. $\boldsymbol{g}'(t) = -4sin(4t)\mathbf{i} - \frac{1}{2-t}\mathbf{j}$)

   - $\boldsymbol{f}(t) \,.\, \boldsymbol{g}(t)$

     (Ans. $-4t^2\sin(4t) + 2t\cos(4t) - \frac{t}{(2-t)(1+t)} -$

     $\ln(1+t)\ln(2-t) + \frac{2t\sin t - 2(1-\cos t)}{4t^2}$)

   - $\boldsymbol{f}(t) \times \boldsymbol{g}(t)$

     (Use $\left(\boldsymbol{f}(t) \times \boldsymbol{g}(t)\right)' = \boldsymbol{f}'(t) \times \boldsymbol{g}(t) + \boldsymbol{f}(t) \times \boldsymbol{g}'(t)$)

   - $\boldsymbol{f}'(t) \times \boldsymbol{g}'(t)$

     (Use $\boldsymbol{f}''(t) \times \boldsymbol{g}(t) + \boldsymbol{f}(t) \times \boldsymbol{g}''(t)$)

   - $\boldsymbol{f}(t^2 + e^{-t})$

     (Ans. $\mathbf{2}(\mathrm{t}^2 + e^{-t})\,\mathbf{i} - \ln(\mathbf{1} + t^2 + e^{-t})\,\mathbf{j} +$

     $\frac{\boldsymbol{2t\, sin\,(t^2+e^{-t}) - 2\left(1 - cos\,(t^2+e^{-t})\right)}}{\boldsymbol{4(t^2+e^{-t})^2}}(2t - e^{-t})\mathbf{)k}$

4. Find the second derivative of the following:
   - $(t^2 e^t \mathbf{i} - t^3 \mathbf{j}) \times (\mathrm{t}e^{-2\mathrm{t}}\mathbf{i} + \mathrm{t}^2\mathbf{j})$
   - $(ln\ t\ e^t \mathbf{i} - t^2 \mathbf{j})\ .\ (\mathrm{t}\,\mathbf{i} + \mathbf{j})$

   (Use product rule for finding the derivative)

5. Find the unit tangent vector at the indicated point and parameterize the tangent line at the indicated point.

$$\mathbf{r}(t) = \cos \pi t\ \mathbf{i} + \sin \pi t \mathbf{j} + t\ \mathbf{k} \quad \text{at } t = 3.$$

(Ans. T(3) = $\frac{-\pi j+3k}{\sqrt{\pi^2+9}}$ and $\mathbf{r}(t) = -\mathbf{i} + 3\mathbf{k} + t\left(\frac{-\pi j+3k}{\sqrt{\pi^2+9}}\right)$)

6. Find the unit tangent vector and the principal normal vector at the indicated point and parameterize the tangent line and the normal at the indicated point.

$$\mathbf{r}(t) = \mathrm{t}\ \mathbf{i} + \mathrm{t}^2\ \mathbf{j} + t^3\ \mathbf{k} \quad \text{at } t = 1.$$

   Also, find the equation of the osculating plane.

(Ans. $\mathbf{T}(1) = \frac{1}{\sqrt{14}}(\mathbf{i} + 2\mathbf{j} + 3\ \mathbf{k})$, **N**(1) = $\frac{1}{\sqrt{266}}(-11\ \mathbf{i} - 8\ \mathbf{j} + 9\mathbf{k})$,

Osculating plane: 3x − 3y + z = 1)

7. Find the point at which the curves intersect and find the angle of intersection.

$$\mathbf{r_1}(t) = \mathrm{e}^{3\mathrm{t}}\ \mathbf{i} + 4\sin(t + \tfrac{1}{2}\pi)\,\mathbf{j} + (t^2 - 1)\ \mathbf{k}$$

$$\mathbf{r_2}(u) = \mathrm{u}\ \mathbf{i} + 4\,\mathbf{j} + (u^2 - 2)\ \mathbf{k}$$

(Ans. (1, 4, -1))

8. Find the points on the curve **r**(t) at which the curves **r**(t) and **r**'(t) have the opposite direction.

$$\mathbf{r}(t) = 5\mathrm{t}\ \mathbf{i} + (3 + \mathrm{t}^2)\,\mathbf{j}$$

(Ans. $(-5\sqrt{3}, 6, 0)$)

9. Find the unit tangent vector, principal normal vector, and an equation in x, y, z for the osculating plane at the point on the curve corresponding to the indicated value of t.

$$\mathbf{r}(t) = \mathbf{i} + 6\mathrm{t}\,\mathbf{j} + 3t^2\,\mathbf{k} \quad \text{at } t = 1.$$

(Ans. **T**(1) = $\frac{1}{\sqrt{2}}(j + k)$), $\mathbf{N}(1) = \frac{512}{289\sqrt{2}}\left(-\frac{1}{2}j + 15\,k\right)$ and osculating plane is given by $x = 1$).