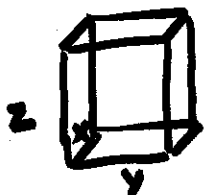


Name: Solution key Lab Section: \_\_\_\_\_

Allowed time: 10 mins

1. A package is in the shape of a rectangular box can be mailed by the USPS carrier if the sum of its length and girth (the perimeter of a cross section perpendicular to the length) is at most  $108 \text{ cm}^2$ . Suppose we want to determine the dimensions of the package that will maximize the volume of the package.

- (a) (3 points) Set up the problem in the form to use the method of Lagrange multipliers, i.e., to maximize  $F$  subject to the constraint  $G = c$ .



Maximize  $F(x, y, z) = xyz$  subject to  
the constraint  $G(x, y, z) = x + 2(y + z)$   
 $= 108 = c$

- (b) (5 points) Determine the system of equations that has to be solved in order to solve the problem. DO NOT SOLVE THE EQUATIONS, JUST FIND THEM.

Lagrange Condition:  $\nabla F = \lambda \nabla G$

$$\begin{aligned} yz &= \lambda \\ xz &= 2\lambda \\ xy &= 2\lambda \\ x + 2y + 2z &= 108 \end{aligned}$$

- system of equations.

2. Classify the critical points  $(1, 2)$  and  $(2, 1)$  for a function  $f(x, y)$  given the following conditions.

- (a) (1 point) If  $f_{xy}(1, 2) \neq 0$ ,  $f_{yy}(1, 2) = 0$ , and  $f_{xx}(1, 2) > 0$  then the point  $(1, 2)$  can be classified as a saddle point.  $[D < 0]$

- (b) (1 point) If  $[f_{xy}(2, 1)]^2 - f_{xx}(2, 1)f_{yy}(2, 1) < 0$  and  $f_{xx}(2, 1) > 0$  then the point  $(2, 1)$  can be classified as a saddle point.

local minimum  $[D > 0, A > 0]$