MATH 21-259 Quiz 6 03/17/11

Name: Solution key Lab Section: Grading key

Allowed time: 12 mins

1. (3 points) Given $f(x,y) = 2xy^2 - 4xy$ where $x = 2st$, $y = st^2$. If we let $F(s,t) = f[x(s,t), y(s,t)]$ then find $\frac{\partial F}{\partial t}(1,-1)$.

$$\frac{\partial F}{\partial t} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial t} = (2y^2 - 4y)(2s) + (4xy - 4x)2st$$

When $s = 1$, $t = -1$ then $x = -2$, $y = 1$

$$\frac{\partial F}{\partial t}(1,-1) = (2-4)(2) + (-8+8)(-2) = \boxed{-4}$$

2. Given $f(x,y) = 2x^2 + 3y$ and point P(1, 1).

   (a) (3 points) Find the directional derivative of f at the point P in the direction of the point Q(5, 4).

$$D_{\hat{u}} f\Big|_P = \nabla f(1,1) \cdot \hat{u} \quad \text{where } \hat{u} = \frac{\langle 5,4\rangle - \langle 1,1\rangle}{|\langle 5,4\rangle - \langle 1,1\rangle|}$$

$$= \langle 4, 3\rangle \cdot \frac{\langle 4, 3\rangle}{5} = 5 \text{ units.}$$

   (b) (4 points)In what direction does f have the minimum rate of change at P? What is this minimum directional derivative of f at P?

Minimum directional derivative is in $-\nabla f(1,1)$ direction.

Note: $-\nabla f(1,1) = -4\hat{i} - 3\hat{j}$. (OR, You can give $-\frac{4}{5}\hat{i} - \frac{3}{5}\hat{j}$ as the answer)

Minimum directional derivative $= -|\nabla f|$

$= -5$ units