MATH 21-259 **Quiz 4** **02/17/11**

Name: ______________________ Lab Section: ______________________

Allowed time: 15 mins

1. Given that $\mathbf{r(t)} = \frac{t+2}{t}\mathbf{i} + \frac{2t}{1+t^2}\mathbf{j}$.

   (a) (1 point) Find the domain of $\mathbf{r(t)}$.

   Domain $= (-\infty, 0) \cup (0, \infty)$

   (b) (2 points) Find $\int \mathbf{r(t)}\ \mathbf{dt}$.

   $\int \mathbf{r}(t)\ dt = \int \frac{t+2}{t}\ dt\mathbf{i} + \int \frac{2t}{1+t^2}\ dt\mathbf{j} = (t + 2\ln t)\mathbf{i} + \ln(1+t^2)\mathbf{j} + \mathbf{C}.$

2. (3 points) Find vector equation of the tangent line to the curve $\mathbf{r(t)} =< t, t^2, t^3 >$ at the point (1, 1, 1).

   Note that $\mathbf{r}'(t) =< 1,\ 2t,\ 3t^2 >$. Thus, the direction vector of the required line is given by $\mathbf{r}'(1) =< 1,\ 2,\ 3 >$. Vector equation of the tangent line: $\mathbf{R}(t) = \mathbf{i} + \mathbf{j} + \mathbf{k} + t(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$.

3. (4 points) Two particles travel along the space curves $\mathbf{r_1(t)} =< t, 3t-2, 4t+1 >$, and $\mathbf{r_2(t)} =< 1+2t, 1+6t, 1+4t >$. Do their paths intersect? If yes, then at what point? Do the particles collide (In other words, do their paths intersect at the same time)?

   First, we set $\mathbf{r_1}(t) = \mathbf{r_2}(u)\ \Rightarrow t = 1+2u,\ 3t-2 = 1+6u,\ 4t+1 = 1+4u.$
   The last equation shows that $t = u$ and first two equation gives that $t = 1+2u$. Solving them together yields us that $t = u = -1$. This means that their paths intersect at the same time which means they do collide. Their paths intersect at the point (-1, -5, -3).