Name: Key Lab Section: ______

1. Let $\mathbf{a} = 2\,\mathbf{i} - \mathbf{k}$, $\mathbf{b} = 2\,\mathbf{i} + 3\mathbf{j} - \mathbf{k}$, $\mathbf{c} = -\mathbf{i} + 3\,\mathbf{j} - 2\,\mathbf{k}$.

   (a) Find a unit vector that is perpendicular to the plane determined by $\mathbf{a}$ and $\mathbf{b}$. (3) points

$$\text{Required vector} = \frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}, \quad \vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & -1 \\ 2 & 3 & -1 \end{vmatrix} = 3\hat{i} + 6\hat{k}.$$

$$= \frac{3\hat{i} + 6\hat{k}}{\sqrt{9+36}} \quad \text{Ans}$$

   (b) Calculate the volume of the "box" that has $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ as sides. (3) points

$$\text{Volume} = |(\vec{a}\times\vec{b})\cdot\vec{c}|$$
$$= |(3\hat{i} + 6\hat{k})\cdot(-\hat{i} + 3\hat{j} - 2\hat{k})|$$
$$= |-3-12| = 15 \text{ units}$$

2. Find $\text{comp}_{\mathbf{b}}\mathbf{a}$ and $\text{proj}_{\mathbf{b}}\mathbf{a}$ given that $\mathbf{a} = \mathbf{2i} + \mathbf{3j} - \mathbf{k}$ and $\mathbf{b} = \mathbf{4i} - \mathbf{3j} + \mathbf{k}$. (4) points

$$\text{Comp}_{\vec{b}}\vec{a} = \vec{a}\cdot u_{\vec{b}} = \vec{a}\cdot\frac{\vec{b}}{|\vec{b}|}$$
$$= \frac{8-9-1}{\sqrt{16+9+1}} = \frac{-2}{\sqrt{26}}$$

$$\text{proj}_{\vec{b}}\vec{a} = (\vec{a}\cdot u_{\vec{b}})\,u_{\vec{b}} = \frac{-2}{\sqrt{26}}\left(\frac{4\hat{i} - 3\hat{j} + \hat{k}}{\sqrt{26}}\right)$$
$$= \frac{-1}{13}(4\hat{i} - 3\hat{j} + \hat{k})$$

Ans