

Name: Key Lab Section: _____

1. Describe the following regions:

(4) points

(a) $\Omega_1 = \{(x, y, z) : |x| \leq 1, |y| \leq 1, |z| \leq 1\}.$

Ω_1 describes the rectangular box with dimensions $2 \times 2 \times 2$ and has a vertex at $(-1, -1, -1)$.

(b) $\Omega_2 = \{(x, y, z) : 1 < x^2 + y^2 + z^2 < 2\}.$

Ω_2 is the set of points that lie inside the sphere of radius $\sqrt{2}$ and centered at $(0, 0, 0)$ and outside the sphere which is centered at $(0, 0, 0)$ and has radius 1.

2. Given $\mathbf{a} = \langle 4, 1 \rangle$ and $\mathbf{b} = \mathbf{i} - 2\mathbf{j}$.

(6) points

a. Find the unit vector with the same direction as the vector $\mathbf{a} - \mathbf{b}$.

$$\vec{\mathbf{a}} - \vec{\mathbf{b}} = \langle 4, 1 \rangle - \langle 1, -2 \rangle = \langle 3, 3 \rangle$$

$$\frac{|\vec{\mathbf{a}} - \vec{\mathbf{b}}|}{|\vec{\mathbf{a}} - \vec{\mathbf{b}}|} = \sqrt{9+9} = 3\sqrt{2}, \text{ unit vector} = \frac{\vec{\mathbf{a}} - \vec{\mathbf{b}}}{|\vec{\mathbf{a}} - \vec{\mathbf{b}}|} = \frac{1}{\sqrt{2}}\hat{\mathbf{i}} + \frac{1}{\sqrt{2}}\hat{\mathbf{j}}$$

b. Find the unit vector with the opposite direction as the vector $\mathbf{a} - \mathbf{b}$.

$$\text{Required unit vector} = -\frac{1}{\sqrt{2}}\hat{\mathbf{i}} - \frac{1}{\sqrt{2}}\hat{\mathbf{j}}$$

c. List all the unit vectors that are parallel to the vector $\mathbf{a} - \mathbf{b}$.

There are two such unit vectors: $\frac{1}{\sqrt{2}}\hat{\mathbf{i}} + \frac{1}{\sqrt{2}}\hat{\mathbf{j}}$ and $-\frac{1}{\sqrt{2}}\hat{\mathbf{i}} - \frac{1}{\sqrt{2}}\hat{\mathbf{j}}.$