Problem set on Lines and Planes MM

1. Find a vector equation that parameterizes the line that passes through the point P(1, 1, 0) and is parallel to the vector $\mathbf{i} - 8\mathbf{j} + \mathbf{k}$. (Ans. $\mathbf{r}(t) = \mathbf{i} + \mathbf{j} + t(\mathbf{i} - 8\mathbf{j} + \mathbf{k})$)

2. Find a vector parameterization for the line that passes through the point P(1, 1, 0) and is parallel to the line

$$\mathbf{r}(t) = (\mathbf{i} - \mathbf{j} + 2\mathbf{k}) + t\,(\mathbf{i} + \mathbf{k})$$

(Ans. $\mathbf{r}(t) = \mathbf{i} + \mathbf{j} + t(\mathbf{i} + \mathbf{k})$)

3. Find the vector parameterization of the line that passes through the points P(1, -2, 3) and Q(2, -3, 4). (Ans. $\mathbf{r}(t) = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k} + t(\mathbf{i} - \mathbf{j} + \mathbf{k})$)

4. Find the scalar parametric equation of the line that passes through the origin and the point P(1, 1, 2). (Ans. $x(t) = t,\ y(t) = t,\ z(t) = 2t$)

5. Find the scalar parametric equation of the line that passes through the origin and is parallel to the line $\mathbf{r}(t) = (\mathbf{i} - \mathbf{j}) + t\,\mathbf{k}$. (Ans. $x(t) = 0,\ y(t) = 0,\ z(t) = t$)

6. Find the scalar parametric equation of the line that passes through P(1, 4, 2) and is perpendicular to the xy-plane. (Ans. $x(t) = 1,\ y(t) = 4,\ z(t) = 2 + t$)

7. Find the set of parametric equations of the line that passes through P(1, 4, 2) and is perpendicular to the plane.

$$P: x + y - z = 2$$

(Ans. $x(t) = 1 + t,\ y(t) = 4 + t,\ z(t) = 2 - t$)

8. Find the symmetric form of a equation of the line that passes through P(1, 1, 2) and is parallel to the line

$$2(x + 1) = 4 (y – 3) = z$$

(Ans. $\frac{x-1}{2} = \frac{y-1}{1} = \frac{z-2}{4}$)

9. Which of the points P(1, 2, 0), Q(0, 0 ,0), R(-5, 1, 5) lie on the line $\mathbf{r}(t) = (\mathbf{i} + 2\mathbf{j}) + t(6\mathbf{i} + \mathbf{j} – 5\mathbf{k})$? (Ans. P lies on the line)

10. Find the points at which the lines

$l_1 : \mathbf{r}(t) = (3\mathbf{i} + \mathbf{j} + 5\mathbf{k}) + t(\mathbf{i} – \mathbf{j} + 2\mathbf{k})$ and

$l_2 : \mathbf{R}(u) = (\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}) + u(\mathbf{j} + \mathbf{k})$ intersect.

Also, find the angle between them.

(Ans. (1, 3, 1)), $\arccos \frac{1}{\sqrt{12}}$)

11. Determine which of the following lines are parallel:

- $l_1 :$ $\mathbf{r}_1(t) = (\mathbf{i} + 2\mathbf{k}) + t(\mathbf{i} - 3\mathbf{j} + \mathbf{k})$
- $l_2 :$ $\mathbf{r}_2(t) = (\mathbf{i} + 2\mathbf{k}) - u(2\mathbf{i} - 6\mathbf{j} + 2\mathbf{k})$
- $l_3 :$ $\mathbf{r}_3(t) = (\mathbf{i} - 2\mathbf{k}) - v(-\mathbf{i} + 3\mathbf{j} - \mathbf{k})$
- $l_4 :$ $\mathbf{r}_4(t) = (1+\frac{1}{3}w)\mathbf{i} + (2 - w)\mathbf{j} + (-3 +\frac{1}{3}w)\mathbf{k}$

(Ans. All)

12. **Find the** distance from a point P(0, 1, 2) to the line $\mathbf{r}(t) = \mathbf{i} + t\mathbf{j}$. (Ans. $\sqrt{5}$)

13. Find the point of intersection of the lines

$l_1 : x(t) = 1 + t, y(t) = -1 - t, z(t) = -4 + 2t$ and

$l_2 : x(u) = 1 - u, y(u) = 1 + 3u, z(u) = 2u$ (Ans. u = -1, t = 1)

14. Find the distance from a point P(0, 1, 2) to the line that passes through the point Q(0, 0, 0) and is parallel to the vector $2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$. (Ans. 2 units)

15. How many unit normals are there for any given plane?

(Ans. Infinitely many)

16. Find the unit normals for the plane $3x - 4y + 12z + 8 = 0$.

(Ans. $\pm\frac{3i-4j+12k}{13}$)

17. Find the angle between $2x + 2y = 3$ and $x + z = -1$. (Ans. $\frac{\pi}{3}$)

18. Given the planes $4x + 4y - 2z = 3$ and $2x + y + z = -1$, show that they are not parallel.

19. Find a set of scalar parametric equations for the line formed by the two intersecting planes $4x + 4y - 2z = 3$ and $2x + y + z = -1$.

(Ans. $x(t) = 6t,\ y(t) = \frac{1}{6} - 8t,\ z(t) = -\frac{7}{6} - 4t$)

20. Determine whether the vectors are coplanar

$4\mathbf{i} - \mathbf{k}$, $\mathbf{i}$, $\mathbf{i} + \mathbf{j} + \mathbf{k}$. (Ans. not coplanar)

21. Find an equation of a plane that passes through the points

$P_1(0, 0, 0)$, $P_2(1, 0, 0)$, $P_3(0, 1, 0)$. (Ans. $z = 0$)

22. Find the distance from the point P(1, 2, 1) to the given plane

p: 2x + y – 4z = $\sqrt{21}$. (Ans. 1 unit)

23. An equation of a plane that intersects the coordinate axes at x = 1, y = -2, z = 3 is given by

(a) $x + \frac{y}{2} + \frac{z}{3} = 0$

(b) $x - \frac{y}{2} - \frac{z}{3} = 0$

(c) $6x + 3y + 2z = 0$

(d) $6x - 3y + 2z = 0$

(e) None of the above. $(x - \frac{y}{2} + \frac{z}{3} = 1)$

24. Find the point of intersection of the surface $x - y + z = 2$ and the curve

$$r(t) = < 1 + t, 2 + t, 3 + t >.$$

(Ans. (1, 2, 3))