<u>Last time</u> <u>Day 15</u>

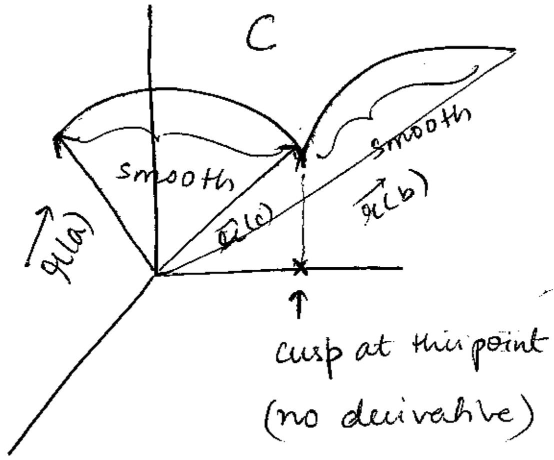
Given a curve $\vec{r}(t) = x(t)\,\hat{i} + y(t)\,\hat{j} + z(t)\,\hat{k}$ where x', y', and z' are continuous fns of t on $[a,b]$. We define

$$\underset{\substack{\uparrow \\ \text{Smooth}}}{L(C)} = \int_a^b \sqrt{x'(t)^2 + y'(t)^2 + z'(t)^2}\,dt$$

Arc length of C

$$= \int_a^c |\vec{r}\,'(t)|\,dt + \int_c^b |\vec{r}\,'(t)|\,dt$$

No derivative at c



Thus, we have def of arc length for piecewise smooth.

Piecewise smooth functions as well.

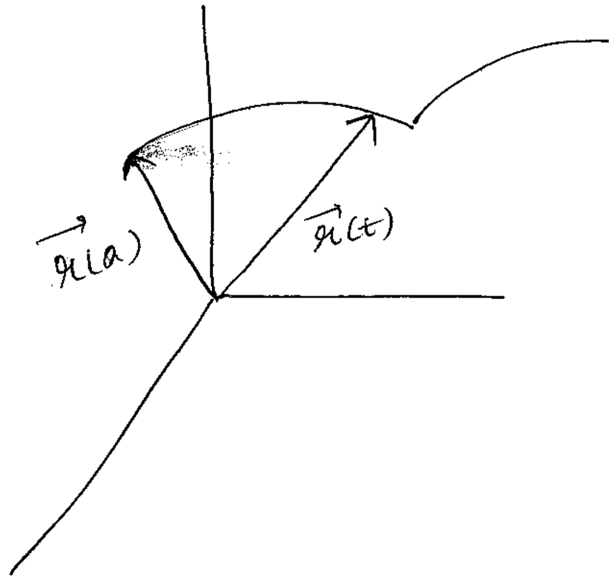
<u>Ex 1</u> Find the length of the arc of the circular helix with vector equation $\vec{r}(t) = \cos t\,\hat{i} + \sin t\,\hat{j} + t\,\hat{k}$ from the point $(1,0,0)$ to the point $(1,0,2\pi)$.

<u>Ex 2</u> Find the length of the arc of the circular helix with the vector equation $\vec{r}(t) = \cos e^t\,\hat{i} + \sin e^t\,\hat{j} + e^t\,\hat{k}$ from the point $(1,0,1)$ to the point $(1,0,e^{2\pi})$.

Remark: Different parametrizations of the same curve gives rise to the same length of the curve. Arc length is just related to the shape of the curve.

Arc length function $\longrightarrow$ moving final point

$$\boxed{s(t) = \int_a^t |\vec{r}\,'(u)|\,du}$$



$$\boxed{s'(t) = |\vec{r}\,'(t)|} \quad \text{FTC}$$

Ex3 Find arc length function for the curve $\vec{r}(t) = \cos t\,\hat{i} + \sin t\,\hat{j} + t\,\hat{k}$ starting at $(1,0,0)$.

Sol^n $$s(t) = \int_0^t |-\sin u\,\hat{i} + \cos u\,\hat{j} + \hat{k}|\,du$$

$$\boxed{s(t) = \sqrt{2}\,t}$$

$s(2\pi) = 2\sqrt{2}\,\pi$ ✓ Ans of Ex1

$$t = \frac{s}{\sqrt{2}}$$

Reparametrize the helix using arc length: (Natural way to parametrizing in terms of arc length)

$$\vec{r}(s) = \cos\frac{s}{\sqrt{2}}\,\hat{i} + \sin\frac{s}{\sqrt{2}}\,\hat{j} + \frac{s}{\sqrt{2}}\,\hat{k}, \quad 0 \le s$$

Sometimes arc length is to used to parametrize the curve to avoid the confusion of too many ways of parametrizing.

<u>Warning</u>: Difficult to compute arc length.

<u>Ex4</u> $C: \vec{r}(t) = t\hat{i} + t^2\hat{j} + t^3\hat{k}, \quad 0 \le t \le 1$

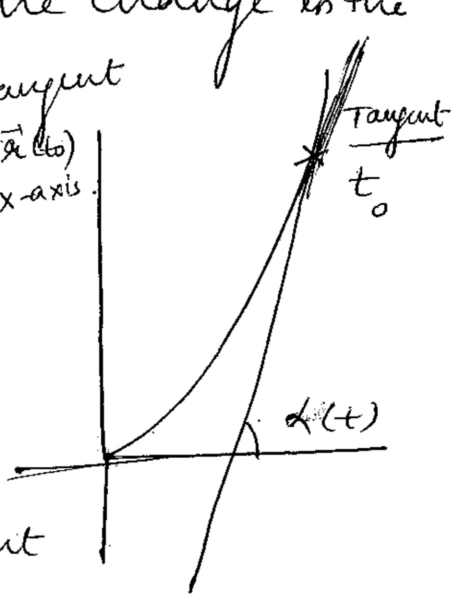
$$L(C) = \int_0^1 \sqrt{1 + (2t)^2 + (3t^2)^2}\, dt$$

$$\boxed{L(C) = \int_0^1 \sqrt{1 + 4t^2 + 9t^4}\, dt}$$

" Not possible to integrate "

Use approximation method to compute this integral

<u>Curvature</u> at any point measures the change in the direction of the curve.



<u>In plane</u>: $\kappa(t) = \left| \dfrac{d\alpha(t)}{ds} \right|$

angle that tangent at the point $\vec{r}(t_0)$ makes with x-axis.

ds → arc length.

<u>In space</u>: tangent slope $\longmapsto$ unit tangent vector.

$\kappa(t) = \left| \dfrac{d\hat{T}(t)}{ds} \right|$ → magnitude — <u>NEXT TIME</u>