Lecture 2 (No lec 1) Day 04

Last time:

Any vector can be represented in terms of its components, i.e. $\langle a_1, a_2, a_3 \rangle$.

Q When do we say that the two vectors are equal?

We say $\vec{a} = \langle a_1, a_2, a_3 \rangle = \vec{b} = \langle b_1, b_2, b_3 \rangle$

if $a_i = b_i$ for every $i = 1, 2, 3$.

Q When do we say that the two vectors are parallel?

We say $\vec{a}$ is $\parallel^{\text{l}}$ to $\vec{b}$ if there is

(shorthand for parallel)

some α [redacted] such that $\vec{a} = \alpha \vec{b}$

i.e., $a_i = \alpha b_i$ for every $i = 1, 2, 3$.

Few Questions

Q1 Find x such that the vector $2\hat{i} + 3\hat{j} + 4\hat{k} \parallel^{le} x\hat{i} + 6\hat{j} + 8\hat{k}$

Sol^n $x = 4$

Q2 Find x such that $\|\hat{i} + \hat{j} + \hat{k}\| = \|x\hat{i} + \hat{j} + \hat{k}\|$

Sol^n $x = \pm 1$ $\left(\because \sqrt{3} = \sqrt{x^2 + 2} \Rightarrow 3 = x^2 + 2 \Rightarrow x^2 = 1 \right)$

Q3 Find a vector that has opp. direction as $\langle 1, 1, 1 \rangle$ but has length 6 unit.

Sol^n I) find a unit vector that points in the direction opposite to $\langle 1, 1, 1 \rangle$

$$\hat{u} = \left\langle \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}} \right\rangle$$

II) Desired vector $= 6\hat{u} = \langle -2\sqrt{3}, -2\sqrt{3}, -2\sqrt{3} \rangle$

Last time (recall)

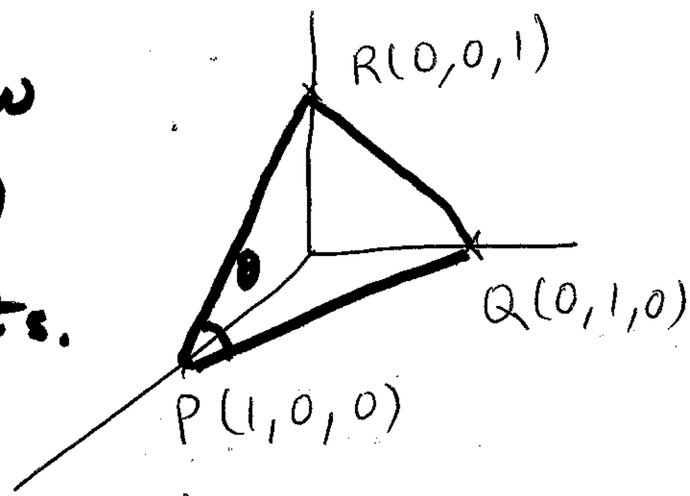
If $\vec{a} = \langle a_1, a_2, a_3 \rangle$ and $\vec{b} = \langle b_1, b_2, b_3 \rangle$

then $\vec{a} \cdot \vec{b} = \sum_{i=1}^{3} a_i b_i = |\vec{a}|\,|\vec{b}| \cos\theta$,

where θ is the angle b/w $\vec{a}$ and $\vec{b}$.

Applications

① Compute angles



Can we find angle b/w the edge $\vec{PR}$ and $\vec{PQ}$?

Yes, using dot products.

"triangle in space"

Prism!

$$\vec{PQ} \cdot \vec{PR} = |\vec{PR}|\,|\vec{PQ}| \cos\theta$$

$$\langle -1, 1, 0 \rangle \cdot \langle -1, 0, 1 \rangle = \sqrt{2}\,\sqrt{2} \cos\theta$$

$$\Rightarrow \quad 1 = 2\cos\theta$$

$$\Rightarrow \quad \boxed{\theta = \frac{\pi}{3}}$$

In general, $\boxed{\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}}$ where

θ is the angle b/w $\vec{a}$ and $\vec{b}$.

<u>Observation</u>

<u>Sign of $\vec{a}.\vec{b}$ is same as the sign of</u> <u>$\cos\theta$</u>

- $\vec{a}.\vec{b} > 0 \iff 0 \leq \theta < 90°$ (acute angle)
- $\vec{a}.\vec{b} < 0 \iff 90° < \theta \leq 180°$ (obtuse angle)
- $\vec{a}.\vec{b} = 0 \iff \theta = 90°$ (right angled)

(Max angle b/w any two vectors is 180°
Min angle b/w any two vectors is 0°)

2. <u>Detect Orthogonality</u> (Greek word)

We say that two vectors are orthogonal if the angle b/w them is 90°, that is, their dot product is equal to zero.

Ex The set of points where $x - y + z = 0$ is

ⓐ an empty set

ⓑ an equation of a line

$\boxed{\text{ⓒ an equation of a plane}}$ ✓

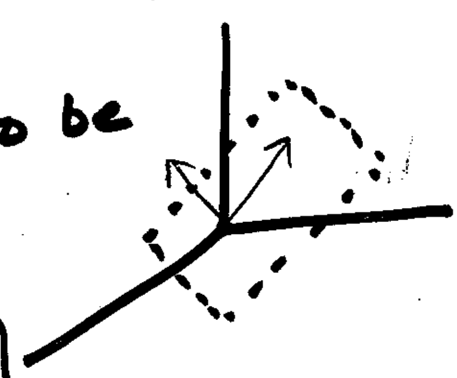
ⓓ hard to solve.

Sol^n why ⓒ is correct?

Note that $x - y + z = 0$ can be rewritten as $\langle 1, -1, 1 \rangle \cdot \langle x, y, z \rangle = 0$

This means that the set of points where $x - y + z = 0$ is same as the set of points which are orthogonal to the vector $\langle 1, -1, 1 \rangle$.

This actually turns out to be a plane.



Think of a vertical vector... all the vectors in the horizontal plane are orthogonal to the vertical vector.

One more application is left that we will discuss next time.

Practice questions.

Find x so that the angle between the vector $\vec{a} = x\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{b} = \hat{i} + x\hat{j} + 3\hat{k}$ is $\frac{\pi}{6}$.