

Problem set on Dot Products

MM

1. Find the value of α for which $(5\mathbf{i} - 2\alpha\mathbf{j} + 2\mathbf{k})$ is perpendicular to the vector $(\mathbf{i} - \mathbf{j})$. (Ans. $\alpha = -5/2$)
2. Find all the numbers x for which $x\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ is perpendicular to the vector $x\mathbf{i} - 3x\mathbf{j} + 20\mathbf{k}$. (Ans. $x = 5, 4$)
3. For $\mathbf{a} = 2\mathbf{i} - \mathbf{j} - 2\mathbf{k}$ and $\mathbf{b} = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$, calculate the cosine of the angle between the two vectors. (Ans. $\sqrt{14}/28$)
4. Find all the numbers x for which the angle between $\mathbf{c} = x\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$ and $\mathbf{d} = 2\mathbf{i} + x\mathbf{j} + 2\mathbf{k}$ is $\pi/3$. (Ans. $x = 0, 8$)
5. Find the angle between the vectors $2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$ and $-3\mathbf{i} + \mathbf{j} + 9\mathbf{k}$. (Ans. $\pi/2$)
6. Find $\text{comp}_{\mathbf{b}}\mathbf{a}$ and $\text{proj}_{\mathbf{b}}\mathbf{a}$ given that $\mathbf{a} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$ and $\mathbf{b} = 4\mathbf{i} - 3\mathbf{j} + \mathbf{k}$.
(Ans. $-\frac{1}{13}\sqrt{26}, -\frac{\sqrt{26}}{13}(4\mathbf{i} - 3\mathbf{j} + \mathbf{k})$)
7. Find the unit vectors that are perpendicular to both $\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ and $3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$. (Ans. $\pm \frac{1}{\sqrt{165}}(8\mathbf{i} + \mathbf{j} - 10\mathbf{k})$)
8. Find the area of the parallelogram determined by the vectors $\mathbf{a} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ and $\mathbf{b} = 4\mathbf{i} - 5\mathbf{j} + 2\mathbf{k}$ using dot products. (Ans. $\sqrt{630}$)
(HINT: Refer to the notes of Jan 19)

9. Find the area of the triangle determined by the points
P(1, 2, 3), Q(4, -5, 2), and R(0, 0, 0). (Ans. $\sqrt{630}$)
10. Use scalar projections to find the distance from a point
P(p, q) to the line $ax + by + c = 0$. (Ans. $\frac{|ap+bq+c|}{\sqrt{a^2+b^2}}$)