

# Directed paths: from Ramsey to Ruzsa and Szemerédi

Po-Shen Loh \*

## Abstract

Starting from an innocent Ramsey-theoretic question regarding directed paths in tournaments, we discover a series of rich and surprising connections that lead into the theory around a fundamental problem in Combinatorics: the Ruzsa-Szemerédi induced matching problem. Using these relationships, we prove that every coloring of the edges of the transitive  $n$ -vertex tournament using three colors contains a directed path of length at least  $\sqrt{n} \cdot e^{\log^* n}$  which entirely avoids some color. We also completely resolve the analogous question for ordinary monochromatic directed paths in general tournaments, as well as natural generalizations of the Ruzsa-Szemerédi problem which we encounter through our investigation.

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\*Department of Mathematical Sciences, Carnegie Mellon University, Pittsburgh, PA 15213. E-mail: ploh@cmu.edu. Research supported in part by NSF grants DMS-1041500 and DMS-1201380, and by a USA-Israel BSF Grant.